ACCOUNT OF THE

LIFE AND WRITINGS

OF

ROBERT SIMSON, M.D.

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ROBERT SIMSON, M.D.

LATE PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF GLASGOW.

By the Rev. WILLIAM TRAIL, LL. D. F.R.S. Edin.

MEMBER OF THE ROYAL IRISH ACADEMY, AND CHANCELLOR OF ST. SAVIOUR'S, CONNOR.

Robertus Simson. M.D.

GEOMETRIAM SUB TYRANNO BARBARO SÆVA SERVITUTE DIU SQUALENTEM,
IN LIBERTATEM ET DECUS ANTIQUUM VINDICAVIT UNUS.

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1812.

TO

THE REVEREND THE PRINCIPAL,

AND TO

THE PROFESSORS,

OF THE

COLLEGE AND UNIVERSITY OF GLASGOW,

THIS ACCOUNT OF THE LIFE

AND WRITINGS

OF A DISTINGUISHED MEMBER

OF THEIR SOCIETY

IS MOST RESPECTFULLY INSCRIBED.

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ADVERTISEMENT.

ABOVE thirty years ago the late EARL STANHOPE honoured me with a requeft to draw up an account of the Life and Writings of the late Dr. SIMSON, of Glafgow, which might be publifhed in the new edition of the *Biographia Britannica*. The flow progrefs of that great work left me much at liberty as to the time of preparing an article which could appear only near the end of it; and for a number of years having been occupied by engagements of a different kind, I was in fome meafure compelled to poftpone the execution of my undertaking, much longer than I wifhed to have done.

As there is not at prefent any near profpect of the completion of the *Biographia*, I could not properly, at my time of life, defer any longer embracing the opportunity afforded me of paying this fmall tribute of refpect to the memory of that eminent man, by whofe friendfhip and inftruction I was honoured during fome of the laft years of his life. I thought it my duty therefore, though the undertaking was ftill

liable to interruptions from other concerns, to collect the neceffary materials, and to arrange them for a feparate publication.

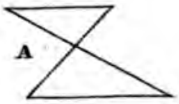
With refpect to the incidents of Dr. SIMSON's life, particularly thofe of the early part of it, I obtained fatisfactory information from a fhort narrative drawn up by the Doctor's colleague and particular friend the late Mr. CLOW, which, from the intimacy fubfifting between them for many years, may be confidered as authentic and accurate. From feveral gentlemen who had been in the fame College with Doctor SIMSON other circumftances have been communicated, and even from my own acquaintance with him, though only for a few years, fome interefting particulars came within the reach of my own obfervation.

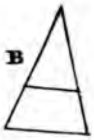
With regard to Doctor SIMSON's fcientific purfuits, there are abundant materials of information in his Works which have been publifhed, and particularly in his learned Prefaces and Notes which accompany them. Some circumftances, alfo, have been collected from his unpublifhed MS. papers, and from the fmall remains of his Correfpondence. It is much to be regretted that the greater part of his Mathematical Correfpondence, which appears to have been very extenfive during a great part of his long life, had been either loft or deftroyed in his own time. Some interefting fragments of it, however, were found among his papers; and a feries of his letters to that eminent mathematician the late Earl STANHOPE, from 1750 to 1758, was in the moft liberal and obliging manner communicated by the prefent Earl; and from them fome curious notices of Dr. SIMSON's ftudies have been extracted.

Some hiſtory and explanation of the ancient geometrical Analyſis, almoſt neceſſarily, forms a part of an account of the literary life of Dr. SIMSON, by whom that Analyſis has been completely reſtored and illuſtrated. The *Mathematical Collections* of PAPPUS, in which is contained nearly every thing that is known from the Ancients of that celebrated inſtrument of inveſtigation, which they employed ſo ſucceſsfully in their geometrical inquiries, from the Doctor's early partiality for that branch of ſcience, naturally became an object of his particular ſtudy. Some account therefore of that work, and of Dr. SIMSON's commentaries on it, is requiſite for giving a juſt view of his geometrical labours, and for eſtimating their importance to, what is ſo much wanted, a corrected edition of that valuable Author. Even with the riſk of ſeveral repetitions, it has appeared convenient to detach from the Memoir the particular account of PAPPUS, and ſome explanations of the ancient Analyſis connected with it, and to annex them as an Appendix. Theſe details, though unintereſting to many readers, may be acceptable to others; and my object in preparing them will be attained, if they can ſave ſome trouble to thoſe who at any future time may undertake a new edition of the *Mathematical Collections.* Two ſhort paſſages of PAPPUS, concerning the ancient claſſification of geometrical lines, are added in a ſecond appendix, in the original language, with COMMANDINE's verſion of them.

To theſe is ſubjoined Dr. SIMSON's tranſlation of the deſcription of the Poriſms of EUCLID, in the Preface to the ſeventh book of PAPPUS; in which are ſome material improvements of the former tranſlations.

August, 1812.





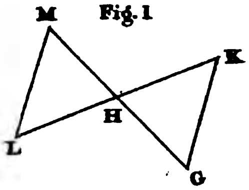
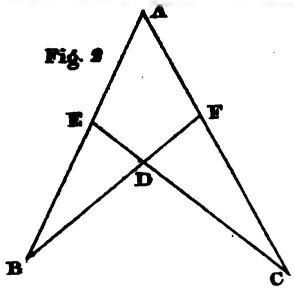


Fig. 1



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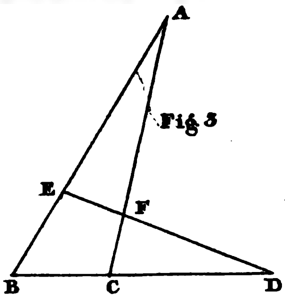

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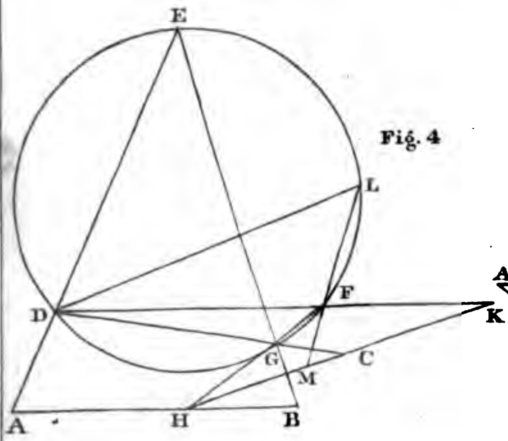

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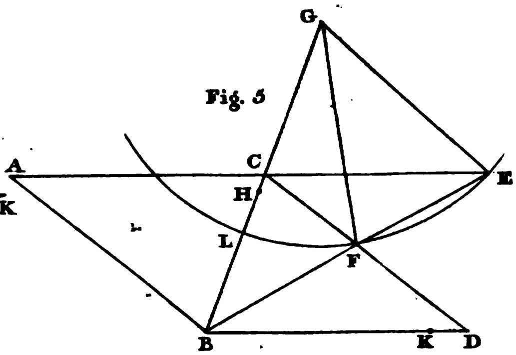

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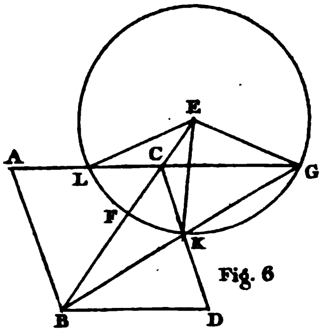

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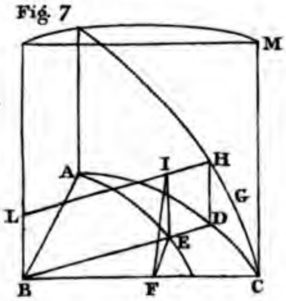

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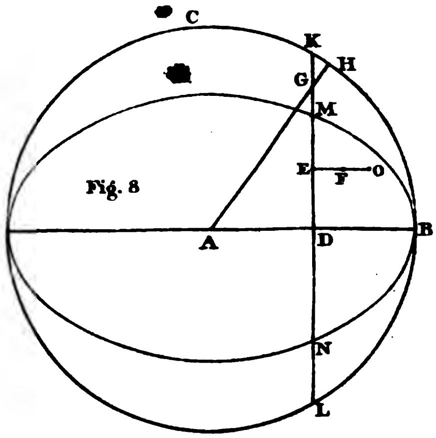

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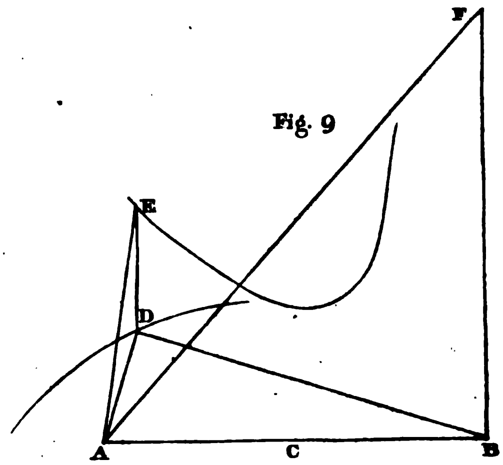

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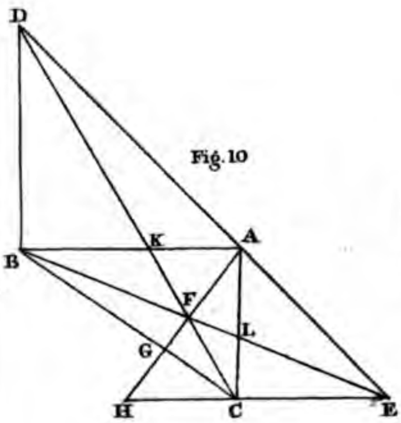

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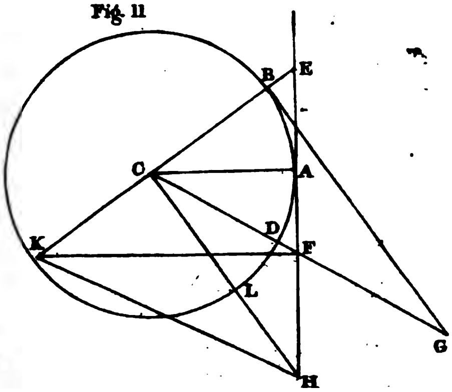

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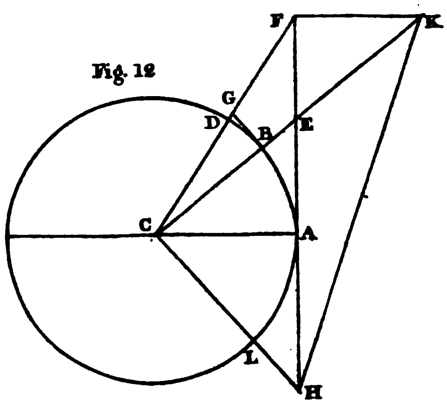

Fig. 12

SECTION I.

General Account of DR. SIMSON'S *Life.*

DOCTOR ROBERT SIMSON, late Profeffor of Mathematics in the Univerfity of Glafgow, was the eldeft fon of JOHN SIMSON, of Kirktonhill in Ayrfhire, and was born on the fourteenth of October 1687, O. S. Being defigned by his father for the Church, after having got the ufual fchool education, he was fent to the Univerfity of Glafgow, where he was diftinguifhed by his proficiency in claffical learning, and in the fciences.

At this time, from temporary circumftances, it happened, that no Mathematical Lectures were given in the College; but young SIMSON'S inquifitive mind, from fome fortunate incident, having been directed to Geometry, he foon perceived the study of that fcience to be congenial to his tafte and capacity. This tafte however, from an apprehenfion that it might obstruct his application to fubjects more connected with the ftudy of theology, was anxioufly difcouraged by his father, though it would feem, with little effect.

Having procured a copy of EUCLID'S Elements, with the aid only of a few preliminary explanations from fome more

advanced ſtudents, he entered on the ſtudy of that oldeſt and beſt introduction to Mathematics. In a ſhort time he read and underſtood the firſt ſix with the eleventh and twelfth books; and being delighted with the ſimplicity of language and accuracy of reaſoning in EUCLID, notwithſtanding the diſcouragements he met with, he persevered in his Mathematical purſuits;* and by his progreſs in the more difficult branches he laid the foundation of his future eminence. But though the bent of his inclination for theſe pursuits was ſtrong, he did not neglect the other ſciences then taught in the College; and in proceeding through the regular courſe of academical ſtudy he acquired the principles of that variety of knowledge, which he retained through life, and which contributed much to the eſtimation of his converſation and manners in ſociety. His chief attention, however, was directed to his favourite ſcience; and his reputation as a mathematician in a few years became ſo high, and his general character ſo much reſpected, that in 1710, when he was only twenty-two years of age, the Members of the College, without any ſolicitation on his part, made him an offer of the Mathematical Chair, in which a vacancy was in a ſhort time expected to take place. From his natural modeſty, however, he felt much reluctance, at ſo early an age, to advance abruptly from the ſituation of a Student to that of a Profeſſor in the ſame College; and therefore he ſolicited permiſſion to ſpend one year at leaſt in London, where, beſides other obvious advantages, he might have opportunities of becoming acquainted

* In ſome of his early MS. volumes ſtill preſerved, are notices of the books he had been reading, and of which he had been making abſtracts. In one of theſe, dated in 1705, (then in his eighteenth year,) are mentioned OUGHTRED's Clavis, RAPHSON's Tracts, JONES's Synopſis, KERSEY's Algebra, and ſeveral others.

with ſome of the eminent mathematicians of England, who were then the moſt diſtinguiſhed in Europe. In this proper requeſt he was readily indulged; and without delay he proceeded to London, where he remained about a year, diligently employed in the improvement of his mathematical knowledge.

This journey turned out very favourable to his views; and he had much ſatisfaction in the acquaintance of ſome reſpectable mathematicians, particularly of Mr. JONES, Mr. CASWELL, and Mr. DITTON. With the latter, indeed, who was then Mathematical Maſter of Chriſt's-Hoſpital, and well eſteemed for his learning, he was more particularly connected; but from all of them he had opportunities of receiving information reſpecting the progreſs of the ſcience, both in England and on the continent of Europe. When the vacancy in the Profeſſorſhip of Mathematics at Glaſgow did occur in the following year, the Univerſity, while Mr. SIMSON was ſtill in London, appointed him to fill it; and the Minute of Election, which is dated March 11th, 1711, concluded with this very proper condition: "That they will admit the ſaid Mr. ROBERT "SIMSON, providing always, that he give ſatisfactory proof of "his ſkill in mathematics previous to his admiſſion." He returned to Glaſgow before the enſuing ſeſſion of the College, and having gone through the form of a trial, by reſolving a geometrical problem propoſed to him, and alſo by giving "a "ſatisfactory ſpecimen of his ſkill in Mathematics, and dex-"terity in teaching Geometry and Algebra;" having produced alſo reſpectable certificates of his knowledge of the ſcience from Mr. CASWELL, and others, he was duly admitted Profeſſor of Mathematics, on the 20th of November of that year.

Mr. SIMSON immediately after his admiffion entered on the duties of his office, and his firft occupation neceffarily was the arrangement of a proper courfe of inftruction for the ftudents who attended his lectures, in two diftinct claffes. He prepared elementary fketches of fome branches in which there were not fuitable treatifes in general ufe. Among his papers one of thefe ftill remains, a tranflation of the three firft books of L'HOSPITAL's Conic Sections, in which geometrical demonftrations are fubftituted for the algebraical of the original, according to Mr. SIMSON's early tafte on this fubject. There remain alfo fome traces of his collections of Problems from HUYGHENS, GREGORY, and others, in optics and aftronomy, for the ufe of his fcholars; and it appears likewife that foon after his admiffion, he had given fome public Lectures on the Hiftory of Mathematics, before the whole Univerfity, of which feveral happen to be preferved, and are proofs of the extent and accuracy of his learning at the early period of life when they were delivered.

Both from a fenfe of duty and from inclination, he now directed the whole of his attention to the ftudy of mathematics; and though he had a decided preference for geometry, which continued through life, yet he did not devote himfelf to it, to the exclufion of the other branches of mathematical fcience; and in the progrefs of this Memoir it will appear that he was well acquainted with the modern analyfis, particularly as it ftood in the early part of his time. From 1711, he continued near fifty years to teach mathematics to two feparate claffes, at different hours, five days in the week, during a continued feffion (or term) of feven months; befides giving occafional

inſtruction which he was ever ready to communicate to thoſe ſtudents, who wiſhed for more particular explanations of his lectures, or to make further progreſs in the ſtudy of mathematics. Though the duties of a profeſſor ſoon became familiar and eaſy to him, yet they occupied a conſiderable portion of his time, and divided it, ſo as often to interrupt the courſe of his private ſtudies.

His manner of teaching was uncommonly clear, and engaging to young people; and moſt of his ſcholars retained through life an affection and reverence for the Profeſſor. The College of Glaſgow in his time was in great repute both at home and abroad, to which Dr. HUTCHESON, Dr. MOOR, Mr. ADAM SMITH, and himſelf, much contributed. The reſort of ſtudents was great, and almoſt all of them attended Dr. SIMSON's lectures. The knowledge of the elementary branches of mathematics, and of the moſt uſeful applications of them, were thence much diffuſed in the College, and ſome taſte alſo for the ſtudy of the higher branches was excited; but the early age of the greater number of the ſtudents, their ſhort reſidence in College, and the neceſſary appropriation of a conſiderable portion of their time to other ſciences, ſeldom admitted of that long and nearly excluſive cultivation of one particular ſcience, by which alone, eſpecially in mathematics, eminence uſually can be attained. Among Mr. SIMSON's ſcholars, however, ſeveral roſe to diſtinction as mathematicians. Dr. MATTHEW STEWART, who alone has applied the geometrical method of reaſoning to the moſt complex phyſical inveſtigations, by univerſal acknowledgement, is to be named the firſt. Mr.

WILLIAMSON,* a favourite pupil, from whom he had great expectations, died very young. Dr. JAMES MOOR, Greek Professor at Glasgow, and Professor ROBISON,† of Edinburgh, were all well known as mathematicians of superior abilities and attainments.

In the year 1758, Dr. SIMSON, being then seventy-one years of age, found it necessary to employ an assistant in teaching; and in 1761, on his recommendation, the Rev. Dr. WILLIAMSON was appointed his assistant and successor.

The resignation of Dr. SIMSON presented an opportunity to the Principal and Professors of recording in their minutes the affection which they felt for the Doctor, and their high admiration of his genius. A long paper for this purpose was drawn up by his colleagues, and it is expressed with all the warmth of attachment and respect, which it was natural for them to entertain for the father of the College, from whom the University had derived so much honour. Several of them had been his pupils, and all had lived with him in habits of friendship from the time of their becoming members of the Society. It is introduced in the following manner:

"The University Meeting do hereby gratefully, unanimously, " and warmly express to Dr. SIMSON their most cordial thanks

* Mr. WILLIAMSON, afterwards the Rev. Dr. WILLIAMSON, Chaplain to the British Factory at Lisbon, in which station he died.

† In the third and fourth editions of the Encyclopedia Britannica (in the article SIMSON R.) is a sketch of Dr. SIMSON's life, which is known to be from the pen of Professor ROBISON, of Edinburgh; and though, from some accidents not now to be ascertained, there are some errors in dates, and in some small circumstances of the narrative, yet it bears the character of the distinguished ability and knowledge of the writer.

" for his long, faithful, and eminent fervices to the Univerfity, " in the courfe of fifty years; during which very uncommon " period, he has with univerfal applaufe, been Profeffor of " Mathematics here; to the great honour of this Univerfity, " as well as to his own high reputation, which will laft as long " as united elegance and fcience are admired among mankind."

It proceeds to mention generally fome of his moft diftinguifhed inventions, juftly ftating that he was the firft, in modern times, who fully underftood and explained the ancient analytical geometry; that he had already reftored, with fuperior accuracy and elegance, fome of the moft valuable works of the ancient geometers, which for ages had been loft; and that he purpofed to employ the remaining leifure of his life, in compleating and publifhing others, not lefs neceffary for the full illuftration of the nature and ufe of the ancient analyfis: " for the accomplifhment of which valuable defign, (the Minute concludes,) " of fo much importance to the advance-" ment of true fcience, the Univerfity Meeting do wifh and " pray that he may long enjoy the bleffing of a vigorous old " age; and they intreat Dr. SIMSON to be entirely affured, " that they will at all times heartily embrace every opportu-" nity of teftifying to him their gratitude, their affection, " efteem, and veneration."

During the remaining ten years of his life, he enjoyed a pretty equal ftate of good health; and continued to occupy himfelf in correcting and arranging fome of his mathematical papers, and occafionally for amufement, in the folution of problems, and demonftration of theorems, which occurred from his own ftudies, or from the fuggeftions of others. His con-

verfation on mathematical and other fubjects continued to be clear and accurate, yet he had fome ftrong impreffions of the decline of his memory, of which he frequently complained; and this probably protracted, and finally prevented his undertaking the publication of fome of his works, which were in fo advanced a ftate, that with little trouble they might have been compleated for the prefs.* His only publication, however, after the refignation of his office, was a new and improved edition of the Data of Euclid, which in 1762 was annexed to the fecond and corrected edition of the Elements. But from that period, though much folicited to bring forward fome of his other works on the ancient geometry, though he knew well how much it was defired, and though he was fully apprized of the univerfal curiofity excited refpecting his difcovery and illuftration of the Porifms of Euclid, he refifted every importunity on the fubject.†

* The following extracts from Dr. SIMSON's Letters to the Writer of this Memoir mention the impreffion he had of the decline of his memory:

In a letter dated February 11th, 1766.——"As to the publifhing any thing on the "ancient geometry, give nobody any hopes of my doing it. You know my inability "to do any thing that requires fo much thought and application as that would do; "and if it be not by your affiftance in preparing them for the prefs, and in taking "care of the printing, I have fcarce any hopes of doing any thing that way, though I "much defire it." And in another letter of March 7th, 1767, he fays, "I wifh you "had not mentioned any thing that might give expectation of the Porifms from me, as "it is very doubtful whether I fhall be able to publifh any thing either about them, "or any other geometrical fubject; my memory, and confequently any other ability "for fuch things, being fo greatly decayed. However, your kind offer of affiftance "will make me give out fome figures to be cut, though I believe they cannot be "ready againft the time you propofe to be here."

I fhall have afterwards occafion to mention the impreffion of a decline of memory which Dr. SIMSON feems to have felt many years before.

† Sometimes he feems to have entertained ferious thoughts of publifhing the moft important of his remaining works, the Sectio Determinata, and the Porifms; and in

A life like Dr. SIMSON's, purely academical and perfectly uniform, rarely contains occurrences, the recording of which could be either interefting or ufeful. But his mathematical labours and inventions form the important part of his life, and fupply the beft illuftration of his character; and with refpect to them, there are abundant materials of information in his printed works; and fome circumftances alfo may be gathered from a number of MS. papers which he left; and which, by the direction of his executor, are depofited in the Library of the College of Glafgow.* It is to be regretted, that of the extenfive correfpondence which he carried on through life with many diftinguifhed Mathematicians, a fmall portion only is preferved. The greater part of it had been loft or deftroyed in his own time; fo that in 1751, when he was defirous of reviewing fome of his early letters to Dr. JURIN on a particular fubject, he obtained a copy, probably from the archives of the Royal Society, of which Dr. JURIN at the time of the

one of his MS. books, of which the earlieft date is of 1762, there is the following notice. "The title-page, if the book to which it belongs come to be printed, may "be thus:

"APOLLONII PERGÆI
"De Sectione Determinata Libri duo,
"Reftituti."
"Quibus adjecta funt non pauca Porifmata; inter quæ habentur quædam EUCLIDIS,
"quibus Doctrina Porifmatum explicata et reftituta eft."

* By the liberal favour of the Principal and Profeffors, the Writer of this Memoir has had every accommodation for confulting thefe Papers, and alfo the very valuable Mathematical Library which Dr. SIMSON bequeathed to the Univerfity. He has received alfo from them fome material communications refpecting their diftinguifhed Colleague; and he is particularly obliged to Profeffors JARDINE, YOUNG, and MILLAR, for procuring ufeful information, and aiding his refearches.

correſpondence (1723) was ſecretary. Through Dr. JURIN he had ſome intercourſe with Dr. HALLEY; and both at that time, and afterwards, he had frequent correſpondence with Mr. MACLAURIN, with Mr. JAMES STIRLING, Dr. JAMES MOOR, Mr. WILLIAMSON of Liſbon, and more particularly with the Rev. Dr. MATTHEW STEWART; of which there are ſome notices in his printed works, and of which alſo there are ſome remains among his unpubliſhed papers. In the latter part of his life, his mathematical correſpondence was chiefly with that eminent Geometer the late Earl STANHOPE,* and with GEORGE LEWIS SCOTT, eſq; then a Commiſſioner of Exciſe, and well known for his ſcientific attainments; and from Dr. SIMSON's letters to both theſe gentlemen, ſome illuſtration of his opinions and purſuits will be obtained, which ſhall be taken notice of in a ſubſequent part of this Memoir.

* The Doctor's affectionate reſpect for that venerable Nobleman is ſtrongly marked by the concern which he expreſſes in a letter to himſelf on hearing a report of his being indiſpoſed. "May it pleaſe the gracious GOD to preſerve ſo valuable a "life for many good purpoſes, both with reſpect to your Lordſhip's honourable family, "and the public good; and particularly for promoting of real and uſeful knowledge "of every kind."——Letter dated 7th March 1755.

SECTION II.

Particular Account of Dr. SIMSON's *Mathematical Studies, and of the Works published by himself.*

IT has already been obferved, that Dr. SIMSON directed his early attention to the ancient Geometry. Dr. HALLEY, by his edition of the Conics of APOLLONIUS, and more particularly by his publication of two Treatifes of APOLLONIUS, accompanied with a corrected edition of the preface to the feventh Book of PAPPUS, gave confiderable aid to thofe who embarked in that ftudy. The *Mathematical Collections* of PAPPUS are indeed the chief repofitory of information refpecting the Geometry of the Ancients, and efpecially refpecting their analyfis, which has been the fubject of much difcuffion among the moderns. This moft interefting work contains fome curious mathematical hiftory of former times, but is more particularly valuable, by the account, contained in the preface to the feventh Book, of the treatifes of the analytical geometry of the ancients, which together obtained the name of *τόπος ἀναλυομένος*. In this preface there is, firft, a general expofition of the analyfis employed by the ancients, both in the folution of problems, and in the demonftration of theorems; then follows a particular defcription of the nature and contents of a certain number of thefe treatifes, which we may prefume

were confidered by PAPPUS as the moft important; and an enumeration of the whole is added, confifting of thirty-three books. The feventh book of PAPPUS itfelf confifts of a number of Lemmata, or fubfidiary propofitions, not contained in EUCLID, but affumed or employed in the feveral treatifes which are fo fully defcribed in the preface.

It is through the medium of COMMANDINE's tranflation only, that the importance of this work of PAPPUS is yet known to the public. Though many manufcript copies of the original remain in various libraries of Europe, it has never been printed; and no attempt has been made to correct the tranflation of COMMANDINE, whofe zeal, learning, and ability, in promoting the knowledge of the ancient Geometricians by ufeful tranflations, are very meritorious; though this pofthumous verfion of PAPPUS, no doubt from its not having received his final corrections, remains in a lefs perfect ftate.*

Dr. SIMSON, however, ferioufly applied himfelf to the ftudy of this tranflation; and notwithftanding its unfinifhed ftate, it was the means of enabling him to inveftigate, and fully to illuftrate the principles, of the analytical geometry of the ancients. The generality of modern mathematicians, from not having fully confidered the intimations of PAPPUS on this fubject, fell into ftrange mifapprehenfions refpecting it.† When they examined the works of the ancient geometers,

* For a particular account of PAPPUS, and of this tranflation, fee the Appendix.

† VIETA muft be excepted; who, though he did not ufe the ancient analyfis in his *Apollonius Gallus*, yet he feems perfuaded, that the ancients had a true geometrical analyfis. For in his firft Appendix to that Treatife (p. 339, Op. VIETÆ) he obferves, "At Algebra quam tradidere, THEON, APOLLONIUS, PAPPUS, et alii veteres analyftæ omnino geometrica eft;" &c.

EUCLID, ARCHIMEDES, and APOLLONIUS, they admired the elegance and clearneſs of their demonſtrations, but wondered by what contrivance they had invented and proved ſo many curious theorems, and obtained ſolutions of ſo many difficult problems. They aſſerted even that the ancients muſt have poſſeſſed an analyſis equivalent to the algebraical; but that they had induſtriouſly concealed it, in order to excite the greater admiration of their inventions. This opinion, naturally improbable, is however unreſervedly expreſſed by ſome eminent mathematicians; by FRANCIS SCHOOTEN, by his brother PETER SCHOOTEN, by PETER NONIUS or NUNEZ; and what is more remarkable, is avowed in later times by the profound Dr. WALLIS,* who was certainly acquainted with PAPPUS, both in the original, and in COMMANDINE's tranſlation. But the important communications of that author reſpecting the geometrical analyſis of the ancients ſeem not to have been duly conſidered by this very eminent Mathematician, though he publiſhed, with learned and valuable notes, a fragment of PAPPUS, not contained in COMMANDINE's tranſlation, but which he diſcovered in one of the two Savilian MSS. of the *Mathematical Collections.*

Dr. BARROW alſo, in his Lectures,§ gives countenance to this erroneous judgment on the ancients, which has been

* WALLIS's Algebra, chap. ii.

§ In Dr. BARROW's four Lectures on the Diſcoveries of ARCHIMEDES, after ſtating his plan in p. 341, he adds, "unde patebit qualem analyſin, et quam noſtræ modernæ "ſimilem exercuerit." And in p. 376, after giving an algebraical analyſis of a problem, (viz. Prop. v. Lib. 2. ARCHIMED. *de Sphæra et Cylindro,*) producing a cubic equation, he derives from it the proportion given by ARCHIMEDES, in which a certain ſtraight line is to be cut in order to reſolve the problem; he adds, "Qui ipſiſſi-

pronounced by so many mathematicians of modern times; and even so late as the publication of Sir ISAAC NEWTON's Fluxions in 1736, Mr. COLSON, the commentator on that work, states this to be the opinion of "many of our modern geo-"metricians."‡

This charge, however, against the ancients, as is justly observed by Dr. SIMSON,§ was altogether groundless. The only algebra known to the ancients was that of DIOPHANTUS,

"mus est analogismus iste, ad quem rem deduxit ARCHIMEDES; quod ipsum satis "prodit et arguit, qualem is analysin usurparit. Nam huc eum devenisse, varias istas "proportionum compositiones, divisiones, permutationes, ac inversiones, quales in dis-"cursu suo ostentat adhibendo, pene supra fidem est." But in this case the ancient analysis is properly applicable; and under the management of ARCHIMEDES, its power in this proposition need not excite surprize. He refers also to the elegant synthetic solution of this problem by HUYGHENS, by means of the trisection of an arch. HUYG. Illustr. Probl. prob. 1. Perhaps some of the discoveries of ARCHIMEDES may have been suggested to him by views and reasonings resembling those of CAVALLERIUS, as is conjectured by his Commentator, TORELLI; but he established their truth by rigorous demonstrations, which might be naturally derived from the only analysis of ancient times, the geometrical.

‡ See COLSON's Commentary on NEWTON's Fluxions, p. 143: and it may be here remarked, that the observations of Sir ISAAC NEWTON, (page 1,) to which this part of the Commentary refers, seem to imply, that he considered the ancient geometry, which he greatly admired, as synthetical only, without having much considered the nature and merits of their analysis. Some of his observations in the *Arithmetica Universalis* naturally lead to the same inference. See sect. iv. cap. 1. art. 18: also, Appendix *de Æquationum Constructione Lineari*, art. 51. Dr. PEMBERTON's account of Sir ISAAC NEWTON's opinion of the ancient geometry, of his regret that he had not studied it more particularly, and also of his remarks on *De Omerique*, corresponds with these observations in Sir ISAAC's works. It may be added, that opinions like that of Mr. COLSON are to be found in many respectable recent writers; and consistently with them, the ancient geometry is generally distinguished as *synthetical*, while the term *analysis* is almost exclusively applied to the modern system.

§ See the Preface to Dr. SIMSON's restoration of the *Loci Plani* of APOLLONIUS, where this opinion is particularly stated, with the Doctor's satisfactory refutation of it.

which was never applied to geometry. But they had an analyfis of their own, of which they made no fecret, and of which there are fome fhort fpecimens, even in EUCLID's Elements.* This analyfis however is more fully defcribed by PAPPUS, who alfo obferves, that in order to facilitate the folution of geometrical problems by this method, the ancients compofed no lefs than thirty-three books, which collected together got the title of τόπος ἀναλυόμενος; and of the chief of which, as has already been mentioned, he gives an interefting account in the preface to his feventh book.

What led probably to this prevailing miftake of modern mathematicians was, that in the moft valuable ancient treatifes ftill remaining, fuch as thofe of ARCHIMEDES, and even the Conics of APOLLONIUS, the analyfis of their propofitions (particularly of their theorems) is generally omitted; though there can be no doubt that this method was employed by them, both in refolving problems, and for afcertaining the truth of theorems, which now appear in their works only in the fynthetic form, and which had been either propofed to them by others, or had occurred to them in their own ftudies. In copying their books, however, for general ufe, for the fake of fhortnefs, (which, before the invention of printing, was an object of confequence,) and even for facility to the reader in acquiring the knowledge of their difcoveries, the analyfis was often omitted. In the fifth book of PAPPUS is an example of this omiffion, for which he affigns the reafon juft now mentioned, and which we may believe influenced the other more ancient geometers. He obferves, that in the com-

* EUCLID's Elem. book xiii. prop. 1, 2, 3, 4, 5.

parifon which he is about to give of the five regular folids having equal furfaces, for the fake of *brevity* and *perfpicuity*, he is to employ only the fynthetic method, and not the analytic, which fome of the ancients had ufed in treating that fubject.† But in the other analytical treatifes of APOLLONIUS, containing folutions of general problems for facilitating the refolution of any particular geometrical problem which can be reduced to a cafe of them, a full analyfis of every cafe is given, being effential to this important application of fuch problems. In the *Sectio Rationis*, recovered from the Arabic by Dr. HALLEY, is a fpecimen of this compleat manner of folution, which, we muft prefume, was followed in the other treatifes, now unfortunately loft.

Thefe books appear to have been all exifting in the time of PAPPUS, but the greater part of them have fince perifhed, or at leaft they are not known to remain, either in the original, or in any tranflation. The book of *Data* by EUCLID, in the Greek; two books of *Sectio Rationis*, by APOLLONIUS, in an Arabic verfion; and feven books of the *Conics* of APOLLONIUS, four in Greek and three in Arabic; are all that have been preferved. But fortunately the defcriptions by PAPPUS of eleven more of thefe books are fo particular and entire, that fome eminent modern mathematicians have been able to reftore them, with various fuccefs indeed, as fhall be afterwards more particularly ftated. Thofe by Dr. HALLEY, from his fuperior tafte and knowledge of the fubject, are in the pureft ftyle of geometry.

† His expreffion is, "*Καὶ τὴν ἔφοδον των ἀποδείξεων ἔχουσας, ὐ διὰ τῆς ἀναλυτικῆς λεγομενῆς* "*θεωρίας δὶ ἧς ἔνιοι τῶν παλάιων ἐποιοῦντο τὰς ἀποδείξεις τῶν προειρημένων σχημάτων, ἀλλὰ διὰ* "*τῆς καῖὰ συνθέσιν ἀγωγῆς ἐπὶ τὸ σαφέστερον καὶ συντομώτερον ὑπ' ἐμῦ, διασκευάσμενας.*" &c. Ex Cod. BULLIALDI, ad fol. 99. a. COM.

The others, by VIETA, SNELLIUS, MARINUS GHETALDUS, FERMAT, SCHOOTEN, and WALLIS, of which ſome are algebraical, and ſome geometrical, though reſpectable works of ingenious men, are defective in that elegance and compleatneſs of ſolution which diſtinguiſh the analytical writings of the ancient Geometers. A field was thus ſtill open, for a perſon of Dr. SIMSON's taſte and genius, for attempting a more perfect reſtoration of theſe curious works of antiquity; and it appears, that ſoon after he was placed in the Mathematical Chair, he ſet about the inveſtigation. From his papers ſtill remaining, we learn that his endeavours were firſt directed to the improvement of the defective reſtorations of these books, by preceding geometers. From the ſame ſource alſo we know, that within a few years he turned his attention to the Poriſms of EUCLID; and it was to be expected that the curious nature of theſe Propoſitions, and even the difficulty of the inveſtigation, would attract the early notice of his ardent mind.

Unfortunately the deſcription of the Poriſms by PAPPUS, in all the MSS. of his *Collections* which have been examined, is ſo mutilated, that every attempt to reſtore them, before Dr. SIMSON's time, had failed. The firſt part of the deſcription, which ſeems to be entire, is calculated only to excite curioſity; being too general for conveying any preciſe notion of theſe Propoſitions, or for giving any effectual aſſiſtance for the recovery of them: and the remainder, containing a detail of the contents of EUCLID's work, is through the whole ſo depraved by the injuries of time, that all endeavours to explain it were nugatory. Some Geometers indeed, of great name, flattered themſelves that they had got

possession of the secret of this peculiar class of Propositions; but subsequent to them, Dr. HALLEY, with all his genius, his extensive knowledge, and his successful experience in unravelling some other pieces of ancient geometry, gives up the Porisms as a hopeless pursuit; and he admits that the description of PAPPUS, as it now stands, is unintelligible and useless.*

Dr. SIMSON, however, fully proved that those ingenious men before Dr. HALLEY, who supposed that they had discovered the Porisms, had certainly deceived themselves; and that the celebrated FERMAT alone, in modern times, had acquired some notion of the nature of Porisms, but without being able to unfold it compleatly; and without having restored any one Proposition, that could be supposed to belong to the Treatise of EUCLID.

The Doctor himself candidly informs us, how long and how seriously he had ineffectually laboured in search of this ænigma, as the nature of a Porism truly became, from the very mutilated state of the only existing description of it. An ardent curiosity was a prominent and well-known part of his character; and in this case, his curiosity was enlivened by his predilection for the study of the ancient geometry; but we may presume also, that a natural and laudable ambition for the distinction which would result from success, in a pursuit in which the greatest Mathematicians of his own and of the preceding age had failed, would animate his zeal and perseverance in the investigation. He had been occupied on this subject even in the year 1715,

* "Hactenus Porismatum Descriptio, nec mihi intellecta nec lectori profutura." See the Preface to the Seventh Book of PAPPUS, edited by Dr. HALLEY, octavo, 1706, page 87, note at the end of the Porisms.

and perhaps before that period; for he obferves, that in that year he had demonftrated the firft cafe of FERMAT's fourth Porifm, before he had acquired the knowledge of the nature of that clafs of Propofitions.*

The firft direction of his refearch feems to have been, to endeavour to difcover the Porifms, from the general defcription of them in the beginning of the account given by PAPPUS; and when this failed, he tried to recover fome of the individual Porifms, from which he hoped to afcertain the diftinctive qualities of thefe Propofitions; but in this attempt he had no better fuccefs. He continued his refearches, and devoted his whole attention to the fubject. For a confiderable time his imagination was compleatly occupied by it: his mind was harraffed by the conftant, but unfuccefsful exertion; he loft his fleep, and his heath was injured: but all his endeavours were ineffectual; and therefore he finally determined to banifh for ever the fubject from his thoughts.

For fome time he maintained this refolution, and applied himfelf to other purfuits;† but afterwards he happened to be walking with fome friends on the banks of the river Clyde at

* SIMSON's Pofthumous Works, p. 540, laft line.

† It is not improbable, that Dr. SIMSON about this time turned his attention to Algebra, and that, on defpairing of the Porifms, he had employed himfelf in the inveftigation of Seriefes for the Circle, of which an account is given in this Memoir, (fee Note H.) The Porifms, as I reckon, were firft difcovered in April 1722. For fome time after, it is reafonable to fuppofe that he was almoft entirely occupied in profecuting his invention; and therefore, as he tranfmitted thefe Seriefes to Dr. JURIN in February 1723, and from his letter alfo it appears that they had been found out fome time before, it is very probable they were inveftigated before his invention of the Porifms. They may be confidered as a refpectable fpecimen of what Dr. SIMSON might have done in Algebra, had he devoted his attention to that branch of Mathematics.

Glafgow, and by accident being left behind his company, he inadvertently fell into a reverie refpecting the Porifms. Some new ideas ftruck his mind, and with his chalk having drawn fome lines on an adjoining tree, at that moment, for the firft time, he acquired a juft notion of one of EUCLID's Porifms.* I have repeatedly heard him relate the occurrence, which he feemed to do with pleafure, and he mentioned even the fcite of the tree on which he defcribed the fortunate diagram.†

After this firft difcovery, however, it required time and much inveftigation, before he could reftore, to his fatisfaction, the general Propofition of EUCLID's firft book of Porifms; and it appears that his firft communication of the difcovery to the Mathematicians of London was through Mr. MACLAURIN. Juft before Mr. MACLAURIN's fetting out from Scotland for France, by the way of London, in 1723, Dr. SIMSON communicated to him the Propofition he had recently reftored, and which a fhort time after was printed in the Philofophical Tranfactions. It appears alfo that Dr. JURIN, then Secretary

* This incident is generally ftated by Dr. SIMSON himfelf, in his Preface to the Porifms. Poft. Works, pp. 319, 320.

† Though moft of Dr. SIMSON's notes and propofitions are dated, yet there is no entry to be found among his papers of this particular incident. From the dates of feveral of his firft inveftigations of the Porifms, both in his PAPPUS, and in other MSS. it had, with much probability, occurred in April 1722; for in that month are fome of his firft notices about the Porifms. At the end of one of them he adds, "Hodie hæc "de Porifmatis inveni, R. S. April 25, 1722." Alfo in a note on Prop. 131, lib. vii. PAPPI, he fays, "Poftquam vero ipfa Porifmata, ea velim quæ generali propofitione "in præfatione ad hunc librum feptimum, complexus eft PAPPUS, multo labore at fuc-"ceffu præter quem fperare æquum fuit felici, tandem ex manca et contracta "admodum PAPPI defcriptione inveftigavimus; facilius erit lemmata hifce infer-"vientia, ad priftinum nitorem reftituere. April 27, 1722. ROB. SIMSON."

of the Royal Society, had mentioned in a letter to Dr. SIMSON this communication from Mr. MACLAURIN; for Dr. SIMSON, in a letter to Dr. JURIN, dated Feb. 1, 1723, in reply to one he had juft received from him, fays, "The Propofition of "PAPPUS, which I fhewed Mr. MACLAURIN when here,* is "the general Propofition into which PAPPUS collected the "Porifms of the firft book of EUCLID's Three; which, together "with the more general Propofition immediately fubjoined, "and which are both deficient, (I mean imperfect,) I have, "with no little inveftigation, recovered and demonftrated, "together with fome few of the Porifms; the reft I have not had "leifure enough to try. I mean thofe of the firft book, for as "to thofe of the two others, excepting what may be included "in the fecond of the above-mentioned Propofitions, I believe "it will be extremely difficult for any body to reftore them. If "I underftand you like to fee them, I fhall fend them as foon "as I can get the fcribbles I have about them wrote over fair."†

It would appear that the refult of this letter was an immediate communication of the Propofitions to Dr. JURIN, which, as might be expected, were much approved by thofe eminent Geometers Dr. HALLEY, Mr. MACHIN, and Dr. JURIN himfelf. In the courfe of the fame year the Paper was printed in the Philofophical Tranfactions,‡ and Dr. SIMSON, in a letter

* This is mentioned in the Preface to Dr. SIMSON's Conics, p. vi. 2d edit. See alfo Note A. at the end.

† The principal object of this letter was to communicate to Dr. JURIN fome Seriefes about the Circle, of which notice will be taken afterwards. See alfo Note H. at the end.

‡ Vol. XL. for 1723, p. 330. In the fame volume of the Tranfactions, p. 248, is a Paper by Dr. PEMBERTON, containing fome propofitions about the Rainbow; at the

to Dr. JURIN, of January 10, 1724, expreffes his fatisfaction at the reception of his communication in the following manner: "The honour you have done me in printing the Paper I fent "up, in the Philofophical Tranfactions, is what I am very "fenfible of; and you may be fure the approbation any thing "in it has had from fuch good judges as Dr. HALLEY, Mr. "MACHIN, yourfelf, and the other learned gentlemen you "were pleafed to fhew it to, cannot but give me a great deal "of pleafure; and nothing could excite me more to endeavour "to reftore the other Porifms, of which there are the leaft "data: but I find it a very difficult affair, efpecially to "one who is fo flow, as by much experience I find myfelf. "However I fhall at leifure hours, GOD willing, try what I "can do. I defire you may give my humble refpects to Dr. "HALLEY and Mr. MACHIN; and be pleafed to tell the Doctor, "that as to the meaning of the words Ἐὰν ὕπτιυ ἢ παρυπτίυ, "ἢ παραλλήλυ, I never was folicitous about the meaning of "the propofition, but at his defire I have confidered the paf-"fage," &c. Dr. SIMSON then enters into a long and learned difcuffion of the proper meaning of this paffage, of which the refult is fhortly ftated in a note, page 348, Poft. Works.*

beginning of which he obferves, "For the greater brevity, I fhall deliver them under "the form of Porifms; as, in my opinion, the ancients called all Propofitions treated "by analyfis only." This intimation naturally gave fome diffatifaction to Dr. SIMSON, as affuming the appearance of being the firft to announce to the world the nature of the ancient Porifms. In the copy of this volume of the Tranfactions belonging to the College Library of Glafgow, the Doctor wrote a fhort animadverfion on the margin of the page, which concludes with thefe words: "but he (Dr. PEMBERTON) has "entirely miftaken the nature of a Porifm, and his two Propofitions have neither the "form nor matter of Porifms. R. S."

* The reft of the letter may be interefting to fome readers, and is therefore placed in Note C, at the end.

For a few years after his discovery of the Porisms, his mind was much engaged in the further prosecution and illustration of it. It appears, however, from his papers, that at this very time he occasionally applied to other branches both of ancient and modern Mathematics; though, as was to be expected, the chief object of his study was the improvement and extension of his investigation of the Porisms, by recovering more of EUCLID's Propositions, and adding also some of his own, and subsequently also the contributions of a few mathematical friends, to whom he communicated the interesting intelligence of his discovery. There are many indications of his intention of publishing the Porisms; but from various causes he postponed the execution of it, till, in the progress of life, he had acquired so very strong an impression of the decline of his faculties, that he reluctantly gave up the design. The *Treatise on Porisms*, however, with some other valuable tracts, were published a few years after the Doctor's death, at the sole expense of his highly respected and learned friend the late Earl STANHOPE; and by the full explanation of the Porisms contained in that volume, (of which I shall have occasion to give a more particular account,) this hitherto inexplicable portion of ancient geometry was, by the Doctor's perseverance and ingenuity, compleatly restored and explained. It may justly be admitted, that the satisfaction and the pride of invention, in the elucidation of this very difficult branch of ancient science, involved in such obscurity from the depraved state of the only remaining account of it, were properly high; and by liberal and scientific minds, the warmth with which he expresses these feelings will not be disapproved: "Descriptio "autem quam tradit (PAPPUS) Porismatum adeo brevis est et

"obſcura, et injuria temporis aut aliter vitiata, ut niſi DEUS "benigne animum et vires dederat in ea pertinaciter inquirere, "in perpetuum forſan geometras latuiſſent."*

It has been already obſerved, that before Dr. SIMSON obtained any juſt notion of the Poriſms, he had begun to improve the reſtorations of other ancient geometrical treatiſes; which had been attempted by preceding geometers, particularly of the *Loci Plani*, and *Sectio Determinata*,† of APOLLONIUS. To thoſe who have ſome knowledge of this ſubject, it is needleſs to explain the value of theſe two treatiſes; particularly of the former, as containing many elegant Theorems, and as being eminently uſeful in the reſolution of Problems. The *Loci Plani* of APOLLONIUS had been reſtored in a certain manner, both by FERMAT, and by FRANCIS SCHOOTEN. In the poſthumous works of the former very diſtinguiſhed Mathematician,‡ is given a reſtoration, geometrical indeed, but ſynthetical only without analyſis, and deficient alſo in other material points, particularly in the diſtinction of the caſes, and in aſcertaining the determinations; without which no geometrical reſolution can be conſidered as compleat.§

* Opera Reliqua, p. 513.

† The *Sectio Determinata* was not publiſhed in his own time, and I therefore defer any notice of it, till an account be given of the volume of Poſthumous Works.

‡ FERMAT had reſtored this work of APOLLONIUS before 1629, as appears by a letter of his, p. 153 of his Works. But it was printed only in 1679, among his Poſthumous Works. See FERMAT. Oper. Varia, tom. 11. p. 12. FERMAT had not ſeen the Preface to the 7th book of PAPPUS in Greek.

§ The uſe of *Loci* in the reſolution of Problems is very obvious, even to thoſe who are but little acquainted with ancient Geometry. The great importance of the compleat diſtinction of caſes and determinations in thoſe treatiſes of the τόπος ἀναλυ-

The reftoration by SCHOOTEN has fimilar defects; in a few only of the problems an analyfis is given, but one purely algebraical; and he acknowledges in his preface* that his reftoration of the *Loci Plani* was defigned to be an illuftration of the geometry of DES CARTES, by furnifhing proper examples of his method. Though it appears that Dr. SIMSON had begun, at a very early period, to reftore the *Loci Plani* after the ancient model, and though the work was almoft compleated before he publifhed his *Conic Sections* in 1735, yet it was not printed till 1749. What reafons occurred for this delay cannot now be conclufively afcertained; but we learn from his papers, and from fome remains of his correfpondence, that at one time he had defigned to add one or two books of *Loci* to thofe of APOLLONIUS. There are many detached Propofitions, and even fome Seriefes of Propofitions of that defcription, in which the two books of APOLLONIUS are quoted as part of the fame work, which fufficiently afcertains his purpofe, at the time of writing them.† It is indeed much to be regretted, that he did not purfue this

ομίνος, which confift of general Problems, to which other Problems may often be reduced, will be explained afterwards. The fame attention to the cafes and determination of *Loci*, and alfo of Porifms, is equally neceffary, to render the application of them ufeful to the folution of Problems.

* F. SCHOOTENII *Exercitationes Mathematicæ*, Præf. 1657.

† In a letter to Earl STANHOPE, dated September 10, 1750, after mentioning that the *Loci Plani* were juft printed, and about to be publifhed, he adds, "that he once "defigned to have added feveral other *Loci Plani*, but thought it beft now to give "only thofe mentioned by PAPPUS, with a few to make fome of them more com- "pleat."——There is alfo among his papers a fketch of a title for the *Loci Plani*, comprehending fome additions to thofe of APOLLONIUS.

ſcheme; as no work could be more generally uſeful in geometrical inveſtigations, than an extenſive and well-arranged collection of *Loci*. The Doctor's plan ſeems alſo to have extended to ſolid *Loci*, of which indeed he gives a hint, in the preface to his *Conic Sections*; and among his papers are alſo ſome ſmall Sets of ſuch Propoſitions, which indicate the probability of the deſign which I ſuppoſe him to have entertained.* The heſitation about making additions to the Treatiſe of APOLLONIUS probably contributed to the delay in printing the work, which was not executed till 1749. He then met with ſome unexpected difficulties in treating with a bookſeller for the ſale of the whole impreſſion, which alone prevented the publication at that time;† and except a few copies diſtributed among his friends in 1750, the book remained unpubliſhed, till after his death. Such is the elegance of method, and the ingenious contrivance of demonſtration in this work, that he has truly exhibited a copy, or at leaſt ſo very nearly a copy, of the work of APOLLONIUS, that little regret need be had for the loſs of the original. The preface alſo is well deſerving the attention of thoſe who wiſh to acquire juſt notions of the ancient books of analyſis.

The firſt publication by Dr. SIMSON, except the Paper on Poriſms in the Philoſophical Tranſactions, was his Treatiſe *Sectionum Conicarum, libri v*, which appeared in 1735.

* In a great many of the Propoſitions of ſolid *Loci* the correſponding algebraical equations are added. From ſome obſervations in the preface to the ſeventh book of PAPPUS, it appears that the ancients had conſidered ſeveral varieties of *Loci*, particularly the plane, the ſolid, the linear, and thoſe called *Loci ad medietates*, ariſing from mean proportionals; but of theſe laſt only a very few imperfect notices remain in the beginning of the third book.

† This account is given by Dr. SIMSON, in a letter to the late Earl STANHOPE, Sept. 10th, 1750.

He had observed, in the first years of his study of Mathematics, that the Treatises on Conic Sections, then in most general use and estimation, were entirely algebraical; and the great merit of the work, written in that stile by the Marquis DE L'HOSPITAL, contributed not a little to the popularity of this mode of treating geometrical subjects. It occurred therefore to Dr. SIMSON, that a treatise on Conic Sections, written on the purer model of antiquity, might have some influence in correcting the prevailing false taste, of introducing algebraical calculation into those branches of geometry where it was not necessary, and where it supplanted a more elegant form of analysis and demonstration. To exhibit, therefore, a just comparison of the two methods, he assumed the same definitions of the Conic Sections, as L'HOSPITAL and others before him had employed; and from them, with the true simplicity and accuracy of the ancient school, he deduced not only the properties of these curves as given by all preceding writers, but added many new and important Propositions of his own,* with the generalization and improvement of many, which had been previously discovered. It is unnecessary to enter into any particular discussion of

* The precise object which Dr. SIMSON had in view, was the occasion of his adopting the definitions of the curves given by L'HOSPITAL and others, in which the language is rather mechanical, than of a strictly scientific character. This I have frequently heard the Doctor express; and he observed at the same time, that he considered the derivation of the properties of these curves from the cone, after the original method of the ancients, as the best; and that if a definition of them, from a description in a plane, were to be assumed as most expedient, a more correct form of language than L'HOSPITAL's ought to be used; which indeed is generally done by those modern geometers who have relinquished the consideration of the cone, in defining this class of curves.

the merits of a work, which has been for ſo many years before the public, which was received with general approbation on its appearance, and which, notwithſtanding more recent improvements, ſtill maintains the reputation, of being the beſt example of the ancient method of demonſtration, of having communicated large additions to the theory of the Conic Sections as it ſtood in his time, and of exhibiting a clear and juſt arrangement of the properties of theſe curves.

This Treatiſe became a part of Dr. SIMSON's Courſe of Lectures in the College; it was reprinted in 1750 with ſeveral additions, including ſome valuable communications by Dr. MATTHEW STEWART; and the three firſt books have been tranſlated, and repeatedly printed, as an elementary introduction to this branch of ſcience. In the preface, Dr. SIMSON gives a ſhort but ſatisfactory ſketch of the hiſtory of this portion of geometrical ſcience from the age of MENÆCHMUS, reputed the firſt inventor, to his own time; in which ſome omiſſions have been remarked, but not of much importance in ſo ſhort an abſtract, as it was his purpoſe to communicate.*

To the ſecond edition of the *Conics* is added an appendix, containing two Geometrical Problems, with a preliminary *Locus*; which Dr. SIMSON gives as examples of the ſuperiority of the ancient analyſis over that of the moderns, in the reſolution of the ſame problems, by the authors there quoted. That ſuperiority will be generally admitted; but it is proper to remark, that beſides the ſolution of one of

* See Note A. at the end.

the problems by GUISNEE,† referred to by Dr. SIMSON, this author, in the same Treatife, gives another very elegant conftruction of the same problem, of trisecting the arch of a circle, by means of two local equations.* The Doctor's object, however, was to ftate a comparifon between the pure geometrical analyfis, and the ufual algebraical method, of refolving an equation, and conftructing that folution. The other method by the combination of *Loci*, though clothed in an algebraical drefs, Dr. SIMSON would have confidered, and with reafon, as in effect geometrical; for the *Loci* in queftion might have been eafily deduced geometrically, and thence the proper geometrical folution became obvious.‡

In the year 1752, Dr. SIMSON tranfmitted to the late Earl STANHOPE, in return for fome valuable communications from his Lordfhip, an inveftigation of a rule of approximation to the roots of numbers which are not perfect fquares, given by ALBERT GIRARD in 1629, without demonftration, in his

† GUISNEE *Appl. de Algebre à la Geometrice*, p. 191, 2d ed.

* The fame Problem is refolved with great fimplicity by BOSCOVICH, by means of an Hyperbola, and the circle to which the given arch belongs: fee MAKO *de Arithm. et Geomet. Æquationum Resolutionibus*, p. 332. It may not be improper to mention here a remark by Dr. SIMSON, in a letter to the late Earl STANHOPE, refpecting a Problem propofed by the Doctor's ingenious pupil Mr. WILLIAMSON of Lifbon. "As to the "fubftituting a circle in place of one of the Hyperbolas, I never tried to do it in this "Problem, becaufe I obferved the ancients were not folicitous about fuch folutions, "and preferred the *Loci* which naturally arife from a Problem to any other, as afford-"ing the fhorteft compofition. Thus in the 58th Prop. of the 5th Book of APOLL. "*Conica*, though it is not difficult to folve the Problem by a circle and the given "Parabola, yet APOLLONIUS takes the Hyperbola that arifes from the Problem as "giving the moft natural and fhorteft folution." The letter is dated March 7, 1755.

‡ Dr. SIMSON's opinions on this fubject are more fully ftated in his correfpondence with GEORGE LEWIS SCOTT, Efq; for which fee Note K. at the end.

edition of the works of STEVINUS. That rule feems to have efcaped the notice of Mathematicians, till Dr. SIMSON undertook the inveftigation of it; which, on the recommendation of Earl STANHOPE, was fubmitted to the Royal Society, and publifhed in the Tranfactions of that year. The method is ingenious, and in fome cafes may be ufeful; though, from modern analytical improvements, fuch rules become of inferior confequence.

The next and only other work of Dr. SIMSON which was publifhed in his life time, was his excellent edition of EUCLID, which appeared both in Latin and Englifh in 1756, and was dedicated to his prefent MAJESTY when Prince of Wales. It contained the firft fix books of the *Elements*, with the eleventh and twelfth, being thofe ufually taught in Univerfities. From his annually lecturing on thefe books, and his accurate knowledge of the ancient geometry, many corrections of the common text of EUCLID would naturally occur to him; but it was only after many and repeated folicitations from his mathematical friends, that he was induced to undertake the preparation of a new edition. Even after he had nearly compleated it, a confiderable delay occurred, from doubts which were then entertained refpecting the conftruction and effect of the Statute of Queen ANNE, for the encouragement of learning. He was flattered by his friends with the hope that an edition corrected by him would have an extenfive circulation, and would nearly fuperfede the common editions then in ufe; and he was therefore naturally anxious to fecure the advantages of his work, at leaft for the term prefcribed by the Statute. But from fome of his letters to the Earl STANHOPE,

it would appear that ſuch were the apprehenſions entertained on this ſubject, particularly about the edition in Latin, that he had it in contemplation to ſolicit a ſecurity of his right by a private Act of Parliament; and the Act in favour of Mr. BUCKLEY, the Editor of *Thuanus*, was quoted as a precedent.* The Speaker ONSLOW was conſulted; but on full conſideration, this ſcheme was relinquiſhed, and Dr. SIMSON relied on the protection of the Statute.

To judge with impartiality of the merits of this work, the ſtate of the text in preceding editions muſt be attended to. Dr. SIMSON, from his veneration for the ancient Geometers, ſeems, with an excuſable partiality, to have aſſumed, that the *Elements* of EUCLID, as they came from the author, were nearly without blemiſh; and he therefore aſcribes all the errors and imperfections of the common editions, either to the careleſſneſs of tranſcribers, or to the blunders of THEON, and other ancient Editors. His corrections are numerous, and many of them important; and even now, when moſt of them are adopted, it might be an uſeful exerciſe for the young mathematician to ſtudy the grounds of his emendations, which exhibit ſo clearly the preciſion of his ideas, and the logical accuracy of his underſtanding. Some animadverſions were made on this edition, chiefly by thoſe whoſe works had been criticiſed in the Doctor's notes; and to ſome of theſe, in a ſecond edition, replies and explanations were made; but he had a great averſion to controverſy, and his obſervations on what he had

* This Act was for preventing for fourteen years the importation of any Latin copies of that work; which was not provided for in the Statute of Queen Anne.

proved to be errors or defects in his predeceffors, were never calculated to provoke it.†

Notwithftanding Dr. SIMSON's valuable corrections, there are ftill fome difficulties in the *Elements*, which remain to be cleared up by fome future Editor. The demonftration of the property of parallel lines (29 I. Elem.) is ftill theoretically defective, requiring the admiffion of fome principle, not ftrictly belonging to the clafs of felf-evident truths. It has by fome been fuppofed, that the remedy for this difficulty muft be fought for in a juft definition of a ftraight line. No definition of a ftraight line has yet been found, and none perhaps can be found, from which all the properties affumed in the *Elements* to belong to it, can be rigidly demonftrated. There is manifeftly alfo fome defect in the definition of a folid angle, fince what is given in Dr. SIMSON's, and in all other editions, does not difcriminate the folid angle from a number of plane angles, formed at one point, which may exift according to the definition, but without forming the folid angle intended to be defined. The improvements and corrections of the Fifth Book are alfo important. His obfervation with refpect to folid figures, in the note on Def. 10. XI. Elem. is curious, from remarking an error, which is fo obvious when pointed out, but which had

† Soon after the publication of EUCLID, Sir ANDREW MITCHEL, then the Britifh Minifter at the Court of Berlin, afked Dr. SIMSON's permiffion to prefent, in the Doctor's name, a copy of the Latin EUCLID to FREDERIC III. The Doctor was gratified by the requeft, and tranfmitted to Sir ANDREW MITCHEL a copy of the book, on the page oppofite to the title of which was written the following infcription:

"CESARI in belligerando, PTOLEMÆIS in promovendis artibus et fcientiis, animumque fuum doctrina excolendo merito æquiparandus, Magnus Imperator FREDERICUS Tertius, BORUSSORUM Rex, ut librum hunc benigne accipiat fumma veneratione exoptat. ROBERTUS SIMSON, Editor."

escaped the notice of the many learned and acute Geometers, who had paid much attention to EUCLID'S *Elements*. An observation of a similar kind, and about the same time, was made by Mr. LE SAGE, which is recorded in the History of the Royal Academy of Sciences at Paris for 1756; and another important correction has been more recently made by LE GENDRE, of which a satisfactory history is given by Mr. PLAYFAIR, in the second edition of his *Elements of Geometry*.

The Book of EUCLID'S *Data* was annexed by Dr. SIMSON to a second edition of the *Elements* in octavo, (1762,) with many necessary corrections, and some valuable additional Propositions, both of his own and of his learned friends the late Earl STANHOPE, and Dr. STEWART. This book is one of the thirty-three, composed by the ancient Geometers, for facilitating the resolution of problems by their analysis; and is the most proper introduction to the study and practice of that analysis.* At the end, Dr. SIMSON has added two geometrical Problems, for illustrating the use of some Propositions of the *Data*, which would not be obvious to beginners; and with respect to one of them he observes, that he believes "it would "be in vain to try to deduce the preceding construction from "an algebraical solution of the Problem." The observation was perhaps hastily made, when he was seventy-five years of age; but he plainly had in view the most common method of resolving geometrical Problems by algebra, viz. by deducing a final equation with only one unknown quantity, resolving that equation, and constructing the solution of it; and with respect

* In a letter of Dr. SIMSON'S, in Note K. are some observations on the use of this book.

to this method, the remark would be juſt. An ingenious friend of the Doctor's, the late GEORGE LEWIS SCOTT, eſq; intimated to him that a conſtruction, ſimilar to his, might be derived from the combination of two *Loci*, expreſſed by two equations ariſing in the algebraical ſolution of the problem. But this communication, with the Doctor's reply, is inſerted in a note at the end, which will be more ſatisfactory than any abſtract of it in this place.*

* See Note K.

SECTION III.

Of Dr. SIMSON'S *Posthumous Works.*

THE strong impression which Dr. SIMSON felt of the failure of his memory, having prevented his publishing some important Geometrical Works; the copies of these works, with a large mass of miscellaneous papers, fell at his death, into the hands of his friend and executor Mr. CLOW, the Professor of Logic in the College of Glasgow.†

While Mr. CLOW was deliberating what was most expedient to be done with regard to these papers entrusted to his care, the late Earl STANHOPE, distinguished in his elevated rank by his ingenious cultivation and liberal patronage of the Mathematical Sciences, intimated his design of publishing those works of Dr. SIMSON which he had compleated, with any other pieces, which though unfinished, might without injury to his fame, be given to the public. The munificent proposal was most acceptable to Mr. CLOW; and after some correspondence respecting the selection, a large volume, in the year 1776, was, at his Lordship's sole expense, handsomely printed, under the care of Mr. CLOW, and liberally distributed.

† A considerable portion of these papers consists of various first sketches of his works, published by himself, or since his death.

This volume contains a reftoration of the *Sectio Determinata* of APOLLONIUS, with two additional books by Dr. SIMSON; and a full explanation of the Porifms, with a reftoration of a number of the Propofitions of EUCLID's original work. To thefe two important works are added fome fmaller Tracts, which fhall be taken notice of in the order in which they are placed in the volume.

The two books of *Sectio Determinata*, it was formerly obferved, had engaged Dr. SIMSON's attention at an early period of his life. That treatife indeed, had been reftored by SNELLIUS, but in a very imperfect manner, without the neceffary diftinction of the various fituations of the Points, called (by APOLLONIUS) *Epitagmata*; and without a compleat expofition of the Determinations, which, as is well known, are neceffary to the perfect folution of any Problem; and are more particularly requifite, as has already been obferved, in the books forming the τόπος ἀναλυόμενος, to render them ufeful for the purpofe for which they were compofed and collected.*

In a long and inftructive preface Dr. SIMSON explains the many defects in the Work of SNELLIUS, which need not be here enumerated.† He takes notice alfo of the fubfequent refolutions of fome of the Problems, by ALEXANDER ANDERSONUS;‡ and of thofe likewife contained in the Treatife

* See the Preface to Dr. SIMSON's *Reftoration of the Loci Plani*, pp. 7 and 8; where the ufe of the books of the τόπος ἀναλυόμενος in refolving Problems is briefly but very clearly ftated.

† It is amufing to obferve SNELLIUS blaming PAPPUS for placing the *Sectio Determinata* after the *Sectio Rationis*, in confequence of his own defective method of reftoring the latter treatife by means of the former.

‡ In *Supplemento* APOLLONII *Redivivi*. Paris, 1612.

of Geometrical Analyfis by HUGO DE OMERIQUE; and in the work itfelf he adopts fome Propofitions from thefe performances. In the Doctor's firft attempts to reftore the *Sectio Determinata*, the difficulty was not fo much the giving proper and even elegant folutions of the various cafes of the general Problem, as the finding the accurate diftinction of the *Epitagmata*, the afcertaining the determinations; and chiefly the inveftigation of folutions, which required the ufe of the *Lemmata* affumed by APOLLONIUS, and demonftrated by PAPPUS,* by which the identity of the reftoration with the original work might be recognized. In the preface he obferves, that it was not till 1727,† and after many unfuccefsful attempts, that he obtained fuch folutions of the fifth and fixth Problems of the firft book. The remaining part of the work required much inveftigation, and amidft the various other ftudies in which he was engaged, it was not till above twenty years after, that he compleated the reftoration to his entire fatisfaction, and in fuch a manner as to leave no doubt of its being truly the fame as the original of APOLLONIUS; or fo nearly the fame, as to preclude the occafion of any further enquiry refpecting it.‡

To the reftoration of the work of APOLLONIUS, two other books, containing an extenfion of the general Problem, are

* For a more particular account of thefe *Lemmata*, fee the Appendix.

† *Opera Reliqua*, Præfat. ad *Sec. Determ.* p. v.

‡ In a letter to Earl STANHOPE, Sept. 10, 1750, he mentions, that for more than twenty years he had been endeavouring to find the ufe and application of the *Lemmata* in PAPPUS, in the folution of the cafes of the *Sectio Determinata*, but had not done fo completely till laft fummer. And among his papers there is an intimation of his having juft completed the reftoration of the *Sectio Determinata*, by the ufe of the *Lemmata* of PAPPUS: to which he adds, " a Geometris intacta funt a tempore PAPPI " in hunc ufque diem. 28 Jan. 1749. R. S."

added by Dr. SIMSON, of which it is neceſſary for me ſhortly to ſtate the hiſtory. About two years before his death, when converſing with him on ſome geometrical ſubject, he mentioned that he had compleated the reſtoration of the *Sectio Determinata*. He informed me alſo, that he had written two additional books, a fair copy of which he took out of his repoſitory, and gave me in confidence, with an injunction to publiſh them, or not, according to the reception which his reſtoration of the work of APOLLONIUS might meet with among mathematicians. At that time he was probably thinking of publiſhing the work of APOLLONIUS, but he did not enter into any particular explanation of his intentions.

During his laſt illneſs, and for ſome time before it, I happened to be at a great diſtance from him, and he died without my receiving any further communication on the ſubject. A ſhort time after his death, therefore, I conſidered it to be my duty, after explaining theſe circumſtances, to deliver the manuſcript to Mr. CLOW, his particular friend, to whom he had aſſigned by his will the property and charge of all his papers. An account of this ſupplement being afterwards communicated to the late Earl STANHOPE, his Lordſhip was pleaſed to deſire that it ſhould be annexed to the Doctor's reſtoration of the work of APOLLONIUS.* From the nature of the ſubject, and

* At the end of a former copy of the reſtoration of this work of APOLLONIUS is the following. N. B. "Unicuique Problematum quæ in duobus de *Sectione* "*Determinata* Libris continentur, addenda ſunt tanquam corolloraria in proprijs "locis, Problemata in quibus quadrata vel rectangula, ſunt quadratis vel rectangulis "majora vel minora, dato quam in ratione; vel quorum unum ſimul cum eo quod ad "alterum datam habet rationum, datum eſt. Nam et hæc ſæpius in uſu veniunt." But inſtead of this, the Doctor had afterwards thought proper to expand the Problem in the two additional books juſt mentioned.

the conftant references to the two former books, the additional books are rather uninterefting, efpecially when read without any particular application of them to the folution of Problems. But they have a fimilar utility to that of the former books; and when any geometrical Problems can be reduced to fome particular cafes of this fupplement, it affords alfo an immediate conftruction and demonftration of fuch Problems.*

The next, and certainly the moft important portion of the pofthumous volume, contains Dr. SIMSON's difcovery and illuftration of the Porifms of EUCLID; and as this volume is not in general circulation, it may be a gratification to fome readers, befides the account already given, to add a more particular detail of the hiftory of this curious piece of ancient fcience, and of the Doctor's fuccefsful inveftigation of it, after it had baffled the refearches of all the ingenious men who attempted it before him.

The feventh book of the *Mathematical Collections*, it has been obferved, is the only notice to be found in ancient authors of EUCLID's Porifms; if we except the few fhort and not very fatisfactory obfervations of PROCLUS, who lived, as is fuppofed, within a century after PAPPUS.† The tranfla-

* Since Dr. SIMSON's death two other reftorations of the *Sectio Determinata* have been publifhed. One by W. WALES, in 1772; and another, by GIANNINI, for which fee *Opuscula Mathematica, auctore* PETRO GIANNINI, Parmæ, 1773.—But with refpect to them, it is fufficient to obferve, that independently of the curious information in Dr. SIMSON's valuable preface, the fuperiority of his work, as a reftoration of APOLLONIUS, muft be obvious to the geometrical reader.

† PAPPUS flourifhed as is generally reckoned about the year 400 of the Chriftian Æra, and PROCLUS towards the year 500, and the former is repeatedly quoted by the latter. See SUIDAS, and GERARD. VOSSIUS *de Universæ Matheseos natura et Constitutione, et Chronologia Mathematicorum*.

tion by COMMANDINE of the fix laft books of PAPPUS, (which were all that were found in the MS. ufed by him) was publifhed in 1588.—The attention of the Mathematicians of Europe was foon directed to that work; and it gave rife to the various attempts which were made to reftore the loft works of the ancient Geometers, fo particularly defcribed in the preface to his feventh book. The Porifms of courfe were not neglected; though the very imperfect ftate of the defcription of them by PAPPUS, in all the manufcripts which have hitherto been examined, created peculiar, and apparently unconquerable, difficulties in the inveftigation.

ALBERT GIRARD, a Geometer of eminence in the early part of the 17th century, was the firft who announced the reftoration of the Porifms of EUCLID. In his *Trigonometry* (1629), and alfo in his edition of *Stevinus* (1634,) he ftates his having reftored the Porifms; but in terms fo general, that no precife opinion can thence be formed of his notions on the fubject. But from the firft of thefe intimations Dr. SIMSON reafonably infers, that GIRARD was not acquainted with the true nature of the ancient Porifms; and it appears alfo to be highly improbable, if he had had any fuccefs in recovering fo curious a branch of ancient learning, that he would have concealed it from the world.*

* Dr. SIMSON in his preface juftly obferves, that the fhort notices of Porifms in C. RENALDINUS's work *De Resol. et Comp. Math.* (Bon. 1644) are of no ufe in explaining the ancient Porifms, and in truth have no relation to them. He feems to ufe the term Porifm for expreffing the general rule refulting from an algebraical inveftigation in general terms: fee pp. 280, 1. They are general corollaries, in the common acceptation of that term. SCHOOTEN alfo, in his *Exercitationes Math.* (fect. 24.) ufes the term Porifm for an inveftigation, without any precife object, of a variety of relations among lines drawn in and about a circle, or other geometrical figure. He gives only an example of a circle.

BULLIALDUS is the next author who mentions them in one of his *Exercitationes Geometricæ*, (1657,) but in it he refers to FERMAT as the inventor, who had communicated the discovery in letters to ſome of his friends at Paris; BULLIALDUS, however, was unable to inveſtigate them.* This very eminent and ingenious man FERMAT appears to have been the firſt in modern times, who made any near approach to the diſcovery of the Poriſms. He ſuppoſed that he had aſcertained the nature of theſe propoſitions, and that he had made ſuch progreſs as to enſure a compleat reſtoration of EUCLID's work. In his treatiſe on the ſubject, which was publiſhed only after his death in 1679, he gives five propoſitions (without demonſtrations) as ſpecimens of his invention. Theſe are certainly Poriſms, but none of them belonged to EUCLID's Treatiſe; and this circumſtance, beſides other arguments, juſtified Dr. SIMSON in ſtating that FERMAT had not acquired a correct notion of the nature of a Poriſm.† It is ſufficient to remark that FERMAT's definition of Poriſms

* Dr. SIMSON, on the margin of his copy of this tract of BULLIALDUS wrote the following remark: "Ex hoc Tractatu liquet BULLIALDUM nihil de naturâ Po-" riſmatum intellexiſſe. R. S. Mart. 29, 1739."

† An Eloge on Monſieur FERMAT appeared in the *Journal des Scavans*, Feb. 9, 1665, eight years after the publication of BULLIALDUS's tract, in which the ſuppoſed diſcovery of FERMAT was announced to the world. This Eloge is alſo copied into the volume of poſthumous works publiſhed in 1679 by his ſon; but in the enumeration of his works which it contained, no mention is made of the Poriſms. Some readers may be gratified with FERMAT's definition or deſcription of the Poriſms. "Cum Locum inveſtigamus Lineam rectam aut curvam inquirimus, nobis tantisper " ignotum, donec Locum ipſum inveniendæ lineæ deſignaverimus, ſed cum ex " ſuppoſito Loco dato et cognito, alium Locum venamur, novus iſte Locus Poriſma " vocatur ab EUCLIDE, qua ratione Locos ipſos unam ſpeciem, et eſſe et vocari, veriſ-" ſime PAPPUS ſubjunxit."—FERMATII Opera, vol. ii. p. 118.

(which at the fame time is not very clearly expreffed,) is avowedly derived from the definition of them by the Mathematicians reckoned modern, (νεώτεροι,) in the time of PAPPUS, and which he pointedly cenfures as inadequate. It may therefore be concluded from this authority, that FERMAT's notion of a Porifm was imperfect; more efpecially as by this definition a numerous clafs of Porifms, altogether unconnected with *Loci*, were neceffarily excluded. The matter, however, is rendered perfectly clear by Dr. SIMSON's full expofition of the nature of Porifms; of which he gives a new definition, very different from that of FERMAT, but fortified by accurate reafoning, and the illuftration of many examples, both of EUCLID's Porifms, and of others compofed by himfelf and his friends. The Doctor alfo obferves, that at an early period he had been ftudying FERMAT's Tract on Porifms, and that in 1715 he had difcovered the demonftration of cas. 1. prop. 4. of FERMAT's Propofitions, before he knew any thing of the nature of Porifms. So little could be derived from the labours of that diftinguifhed Geometrician, by Dr. SIMSON, in his inveftigation of the common object of their purfuit.*

Dr. DAVID GREGORY alfo, an eminent Geometer in the latter part of the fame century, feems, from having taken only a fuperficial view of the fubject, to have deceived himfelf, when he obferves, in the preface to his valuable edition of EUCLID, that it would not be difficult to reftore the Porifms of EUCLID, if the Greek text of PAPPUS were publifhed.

* Dr. HALLEY, in the preface to the *Sectio Rationis*, (laft page) fpeaking of a work of FERMAT's, adds, "qui (fc. FERMATIUS) et Porifmata EUCLIDIS opus longe "difficillimum, redintegrare pollicitus eft, verum fidem non liberavit."

That text was within his reach, as there were two Greek manufcripts of PAPPUS in the Savilian Library, of which, as Profeffor of Aftronomy, he had a particular charge; but there is no evidence of his having made any attempt, and he died while engaged in another important work, the publication of the *Conics* of APOLLONIUS, which was finifhed by his Colleague Dr. HALLEY. From the Savilian manufcripts, Dr. HALLEY was enabled to give an improved edition of the preface to the feventh book of PAPPUS, with a translation much fuperior to that of COMMANDINE. The general account of the Porifms is in fome places corrected; but the detail of the contents of EUCLID's work remained fo very defective and unintelligible, that Dr. HALLEY, with his great abilities and learning, and with his fuccefsful experience in reftoring and elucidating feveral works of APOLLONIUS, was compelled to give up the inveftigation of the Porifms as hopelefs; and he was brought to this conclufion, both from the difficulty of the fubject, and alfo from the mutilations which the injuries of time had occafioned in all the manufcripts which had been examined.*

From all thefe feemingly unfurmountable obftructions, it required an uncommon degree of zeal and fortitude in Dr. SIMSON to undertake the inveftigation; and from his own account, which has already been given, his perfeverance was not lefs remarkable. From the time of his firft difcovery of the nature of a Porifm, the Doctor, without neglecting his other fcientific purfuits, occafionally applied his mind to this very curious fubject; and notwithftanding the many difficul-

* See Note B. at the end.

ties which he encountered, he was at length completely ſucceſsful; not only in explaining the diſtinctive character of theſe Propoſitions, but in the inveſtigation of a number of the individual Poriſms of EUCLID, and alſo of various other Poriſms, which were uſeful, both in illuſtrating the nature of theſe Propoſitions, and in ſhewing the application of them in the reſolution of difficult geometrical Problems.

After a certain progreſs in the proſecution of this ſubject, it became an important object to aſcertain a juſt definition of the Poriſm. The definition given by the later Mathematicians, as ſtated by PAPPUS, but cenſured by him, "quod "deficit hypotheſi a Theoremate Locali,"* clearly implies that a Poriſm had an immediate reference to a *Locus*; though it is not leſs clear that PAPPUS conſidered *Loci* as only one claſs of Poriſms, (a large one no doubt,) but that of courſe many Poriſms have no connection whatever with *Loci*.

But the definition which PAPPUS quotes from the ancients,† as more characteriſtic of Poriſms, is too general for any uſeful purpoſe; and though it does correſpond to the nature of theſe Propoſitions, yet it is deficient in diſcrimination, and of itſelf neither conveys any preciſe notion of EUCLID's Poriſms, nor gives aſſiſtance in the inveſtigation of any individual Propoſition.

* "Quod deeſt in hypotheſi Theorematis Localis." HALL. verſio.

† PAPPUS, in his deſcription of the Poriſms, (preface to 7th book,) ſtates the following definitions of a Theorem, a Problem, and a Poriſm: "Differentias autem horum "trium, melius intellexiſſe veteres, manifeſtum eſt ex definitionibus. Dixerunt enim "*Theorema* eſſe quo aliquid proponitur demonſtrandum; *Problema* quo proponitur "aliquid conſtruendum; *Poriſma* vero eſſe quo aliquid proponitur inveſtigandum." HALLEII verſio.

After much confideration of various forms of a Definition which had occurred to him, the Doctor finally fettled the following: "Porifma eft Propofitio in qua proponitur demon-"ftrare rem aliquam, vel plures datas effe, cui, vel quibus, ut et "cuilibet ex rebus innumeris, non quidem datis, fed quæ ad ea "quæ data funt eandem habent relationem, convenire often-"dendum eft affectionem quandam communem in Propofitione "defcriptam."*

The Doctor illuftrates the propriety and accuracy of this definition by many examples; and fhews particularly wherein the definition blamed by PAPPUS coincides with his, and wherein it is deficient, by excluding many genuine Porifms. The definition indeed, with much addrefs, is fo framed as to correfpond with all the intimations of PAPPUS refpecting Porifms, and alfo with the character of the individual Porifms of EUCLID, which Dr. SIMSON had difcovered; and therefore may juftly be confidered as expreffive of the notions on this fubject entertained by the ancients. It is not pretended that

* From the neceffary generality of expreffion for comprehending every clafs of Porifms, there is fome obfcurity in this definition; but the Latin language, which Dr SIMSON ufed in all his mathematical writings, is in this cafe better fitted for giving precifion, and for preventing ambiguity, than the Englifh. In Mr. LAWSON's tranflation of Dr. SIMSON's Introduction to the Porifms, which is plainly meant to be ftrictly literal, this definition is expreffed thus: "A Porifm is a Propofition in which it is propofe "to demonftrate that fome one thing or more things are given, to which, as alfo to "each of innumerable other things, not indeed given, but which have the fame "relation to thofe which are given, it is to be fhewn that there belongs fome common "affection defcribed in the Propofition." The following lefs literal tranflation is propofed by Mr. PLAYFAIR, for remedying part of the obfcurity: "A Porifm is a "Propofition in which it is propofed to demonftrate that one or more things are "given, between which and every one of innumerable other things, not given, but "affumed according to a given law, a certain relation, defcribed in the Propofition, is "to be fhewn to take place." Ed. Tranfact. vol. iii. p. 172, Note.

this was a definition of the ancients; for probably no precife definition was given by them, of either Theorem, Problem, or Porifm. None appears in the works of the more early Geometers, which are ftill preferved in a confiderable degree of purity, and where fuch definitions would naturally have had a place. And we may affirm with much probability, that if any ufeful and characteriftic definition of a Porifm had reached the times of PAPPUS, he would not have neglected fo valuable a remnant of ancient mathematical fcience, in a work obvioufly defigned for the prefervation of the more curious portions of it. He does not omit a definition, which probably was only a traditional and pointed obfervation of fome ancient Geometer; and though of no ufe in explaining the character of a Porifm, yet it in fome degree fortified his objection to the definition of the later Mathematicians, who, he ftates, from inability, could not accomplifh the inveftigation of Porifms; but fatisfied themfelves with affuming the conftructions as they found them in EUCLID, or other Geometers, and adding the demonftrations.

It is obferved by PAPPUS, that a Porifm is neither a problem nor a theorem, but fomething of an intermediate nature; and that it might be propofed either as a problem or as a theorem; fome Geometers contending for the one, and fome for the other. Dr. SIMSON has given a form to the enunciation of a Porifm, implying this intermediate character between a problem and a theorem. In his enunciation it is affirmed that certain things *may* be found, which fhall have the relations or properties therein defcribed. Perhaps this form refembles more that of a theorem than of a problem; but at the fame

time, the things, of which it is faid that they may be found, muft be actually inveftigated by analyfis, as if the Propofition were a problem. Were it fimply propofed to inveftigate certain things which would have the properties expreffed in the Porifm, it may be regarded as a problem; but if thefe things are found by a conftruction defcribed in the enunciation, the Propofition becomes a theorem, affirming the truth of the properties afferted; and then a demonftration only is required, without any inveftigation; in the manner which appears to have been practifed by the later Mathematicians, alluded to by PAPPUS.*

The moft fatisfactory illuftration of thefe definitions is by examples, of which Dr. SIMSON gives a great variety;

* The enunciation of a Porifm as a problem, is not confiftent with the ufual character of fuch Propofitions. Problems commonly (whatever difficulty attend the actual refolution of them) are almoft immediately recognized by thofe having fome knowledge of geometry, as either poffible in certain circumftances of the data, or as altogether impoffible; and it is unufual to propofe as a problem, "to find things with "certain properties, refpecting the poffibility of which no judgment can be formed "without an analyfis, or fuch confideration as is equivalent to an analyfis." For example, if it had been propofed as a problem in the time of APOLLONIUS, to find in a given parabola a point having the property of the focus, that point being then unknown; fuch a Propofition would not have been confidered as a proper problem, but in reality would have been a genuine Porifm.——PROCLUS, in his defcription of Porifms, mentions the 1 Prop. 3 Elem. "to find the center of a circle," as a Porifm, being in fome meafure between a problem and a theorem. But in that cafe, as a circle, from EUCLID's definition of it, muft have a center, the Propofition "to find that center," feems to be a proper Problem. Had the circle been defined from another of its properties, as for inftance, from its being produced by the extremity of a ftraight line moving at right angles to another ftraight line given in magnitude and pofition, and in the fame plane, fo that the fquare of the moving line be equal to the rectangle by the fegments into which it divides the given line; then the finding of the center would be a proper Porifm, and might be enunciated thus: "within a given "circle (defined in the manner now mentioned) a point may be found, from which "all ftraight lines drawn to the circumference will be equal."

and he diftinguifhes clearly the Locus and Local Theorem from the Porifm, though all the three, in a large clafs of the latter, are convertible into each other. He proceeds to give examples of Porifms altogether unconnected with *Loci*, and adds likewife an algebraical or arithmetical Porifm; all of which, however, are clearly comprehended in his own definition, and difplay the defect of the definition cenfured by PAPPUS.

After an ample expofition of the nature of Porifms, the Doctor proceeds to the reftoration of fome of EUCLID's Porifms, beginning with the general Propofition contained in PAPPUS, though in an imperfect ftate, and diftributed into the ten cafes alfo remarked by him; adding the fecond general Propofition, which is an extenfion of the firft. He then inveftigates feveral other Porifms afcertained to belong to EUCLID, from the remaining fragments of his defcription; and by employing fome of the *Lemmata* affumed by EUCLID, and preferved by PAPPUS, he proves the identity of his inveftigation with that of the original author. The Doctor's inveftigation of Porifms, of courfe, is in the ancient analytic method; though from the general nature of thefe Propofitions, fome variation in the form, from what is ufed in a common problem, is requifite.* The things to be inveftigated, as in the common analyfis, are fuppofed to be found; and the relation of them to the innumerable other things muft alfo be affumed, and often, as exifting in different fituations, to exprefs effectually the general nature of the Propofition.—It is unneceffary here, however, to enter more particularly into this difcuffion, as the propriety and ufe of the form adopted by Dr. SIMSON can

can be fully explained only by examples, and will be intelligible to thofe who have duly confidered what he has written on the fubject.*

FERMAT's five Propofitions, though ftated by him in the form of theorems, are undoubtedly Porifms, and as no inveftigation or demonftration was given of them fo far as Dr. SIMSON could learn, they are properly added. He gives them the appropriate form of Porifms, renders them more general, and adds the neceffary analyfis and compofition. The fecond Prop. of FERMAT belongs to the parabola, and is demonftrated in the Doctor's *Conics*, Prop. 19. Lib. v. 2d edit.

The Porifms of EUCLID, fo far as may be conjectured from the defcription of PAPPUS in its prefent imperfect ftate, feem all to have had reference to the circle and ftraight lines, and therefore may be called plane; but it is obvious that the Conic Sections may fupply many folid Porifms, and fuperior lines in like manner may be the fource of fuperior claffes of Porifms, which the ancients would have called linear. Dr. SIMSON concludes this work with fome elegant Porifms of his own, and fome by Dr. STEWART, one of the very few to whom he had, in the earlier part of his life, freely communicated his invention.‡ He takes occafion alfo to fhew the

* I fhall quote here an important obfervation of Dr. SIMSON's on this fubject. *Opera Reliq.* p. 337. "Semel igitur ad minimum, punctum, recta, angulus, vel "quodcunque fuerit de quo in Porifmate aliquid generaliter affirmatur, fumendum "eft utcunque, intra tamen limites, fi qui fint in Porifmate præfcriptos, ut legitime "fiat analyfis. Sæpe autem brevius et facilius ea quæ inveftiganda funt inveniuntur, "fi tum quævis ex prædictis rebus fumatur utcunque, tum denuo modo quodam "particulari fumatur, quando hoc fieri poteft."

‡ See Note, 2 C. at end.

utility of Porifms in producing eafy refolutions of geometrical problems, which, without that affiftance, might be complex and difficult.

This application of Porifms by the ancients is ftated by PAPPUS; who infinuates alfo, that little had been done refpecting them by the geometers fubfequent to EUCLID; but as EUCLID's treatife is part of the τόπος ἀναλυομένος, we may prefume that this ufe of the Porifms was in contemplation of the author,† and was probably regarded by fubfequent geometers as the moft important.

I cannot omit adverting in this place to a very ingenious theory of Porifms propofed by Mr. Profeffor PLAYFAIR, of Edinburgh, firft briefly in his account of the life of Dr. STEWART,* and afterwards more fully explained in a memoir on that fubject in the third volume of the Tranfactions of the Royal Society of Edinburgh. The refult of his inveftigation is, that a Porifm is the cafe of a problem which becomes indeterminate; or more particularly, "a Porifm is a Propo-"fition affirming the poffibility of finding fuch conditions "as will render a certain problem indeterminate, or capable "of innumerable folutions."§ But though I admire the ingenuity, and fully admit the foundnefs, of this definition,

† It is proper to obferve that Dr. SIMSON gives a corrected edition of all the *Lemmata* belonging to the Porifms, which muft be very ufeful to thofe who may attempt to recover more of EUCLID's Porifms.

* This life of Dr. STEWART, read before the Edinburgh Society in July 1784, was publifhed in the firft volume of the Tranfactions.

§ In this paper is given a comprehenfive and very ingenious theory of the nature of Porifms founded on this definition; to which the reader who is defirous of fatisfactory information on the fubject is referred.

and alfo the utility of the principle on which it is founded, in the difcovery of Porifms, I muft acknowledge my doubt of that particular notion of a Porifm having ever been adopted, or even propofed, among the ancient geometricians. The circumftance of its being fo fatisfactory as a definition is to me a proof that it was never generally known or embraced: for had it ever been approved and eftablifhed, it feems fcarce poffible that it fhould afterwards have been neglected and loft. That among the ancients the confideration of the relations fubfifting among the data, in fome problems, might have occafionally fuggefted the particular cafe in which thefe problems would become indeterminate, is very probable. It might alfo have often occurred to them, that this indeterminate cafe involved an important general Propofition, which might be feparately ftated as fuch, and preferved. Many Porifms of EUCLID may poffibly have been invented in that way; but ftill I entertain a doubt, if ever the ancients were in poffeffion of this notion as a principle, and as the proper ground of the definition of a Porifm. PAPPUS mentions the definition of the ancients, and apparently as the only one which they were known to poffefs, though, as has been remarked, it be of no particular ufe. He mentions alfo a definition of the later mathematicians, which he cenfures as erroneous: but if fuch a compleat and fatisfactory definition, which not only accurately diftinguifhes that clafs of Propofitions, but points out an obvious fource of the difcovery of them, had ever been generally underftood among the ancients, it is difficult to fuppofe that it could ever have been loft; and had it reached the time

of PAPPUS, it is moſt improbable that he ſhould neglect the recording of it in his detailed account of EUCLID's treatiſe on this ſubject. With theſe ſtrong internal probabilities, and the total want of external evidence, I muſt (with deference, however, to the opinion of thoſe who may think differently) adhere to the judgment which I have already expreſſed concerning the recent origin of this excellent definition propoſed by Mr. PLAYFAIR.*

The very compleat account of the Poriſms given by Dr. SIMSON in this poſthumous treatiſe may be conſidered as ſo ſatisfactory an illuſtration of the nature and character of EUCLID's work, that the juſtice of the concluding remark of his preface will readily be admitted: "His igitur doc-"trinam Poriſmatum ſatis explicatam, et in poſterum ab "oblivione tutam fore, ſperare liceat." I may take the liberty of adding, that though Dr. SIMSON's fame, as an accurate, elegant, and ingenious Geometrician, be eſtabliſhed from his other works; yet the reſtoration of the Poriſms of EUCLID will be regarded by poſterity as the moſt important production of thoſe powers of inveſtigation and genius, with which he was ſo eminently endowed.

There remain ſome ſmall tracts in the poſthumous volume, which ſhall be very briefly mentioned.

The treatiſe *De Logarithmis* is formed on the model of the fifth book of EUCLID; and notwithſtanding the great variety

* See Note D. at the end. I may here obſerve that the two definitions are perfectly conſiſtent: but though Dr. SIMSON's might eaſily be derived from Mr. PLAYFAIR's, yet the deduction of Mr. PLAYFAIR's from the other is by no means ſo obvious, and does not appear to have ever occurred to Dr. SIMSON.

of explanations which have been given of that celebrated and moſt uſeful invention, yet a deduction of the properties of *Logarithms*, with the ſame ſtrict manner of demonſtration which is uſed by EUCLID in treating of proportion, remained a deſideratum in the ſcience. It was undertaken at the requeſt of the late Earl STANHOPE, who ſuggeſted to him ſome valuable hints on the ſubject; and the tract, though ſhort, is compleat, and appears to be the laſt work which he finiſhed. His correſpondence with the Earl STANHOPE about the plan of it was in the year 1752; and the fair copy from which the tract was printed, is dated Feb. 19, 1762. But in this ſome corrections were made with the ſanction of the Earl STANHOPE.

The next tract, *De Limitibus Quantitatum et Rationum*, is only a fragment; but it contains a rigorous demonſtration of the principles of fluxions, and of prime and ultimate ratios. It is well known that from the time of the invention of fluxions, ſome inaccurate expreſſions reſpecting infinitely little quantities, and ſome looſe reaſonings founded on ſuch expreſſions, had crept into that branch of ſcience. Much diſcuſſion aroſe, and a ſerious controverſy was maintained for ſome time; which, in a ſcience profeſſing to be founded on ſelf-evident principles, and conducted with ſtrict demonſtration, was of itſelf a matter of ſome reproach. Mathematicians of conſiderable name took a part in the buſineſs; and many treatiſes were written, with a view to remove the objections which had been ſtarted, and alſo to give a rigid demonſtration of the principles and rules of this new and moſt important analyſis. The doctrine of limits was certainly implied (though not in language

calculated to prevent every objection) in Sir ISAAC NEWTON's Propositions of prime and ultimate ratios. The principle, however, came afterwards to be stated in more accurate language; and is now by universal consent admitted to be the sound foundation on which this branch of the science ought to rest. This certainly has been properly explained in other treatises; yet it may be hoped that this fragment of Dr. SIMSON's will be considered by the admirers of the simplicity and accuracy of ancient geometrical reasoning, not only as free from even the pretence of objection, but as a demonstration, with elementary perspicuity, of the principles of fluxions.*

At the end of the volume is an Appendix, containing a few geometrical problems resolved in the ancient analytical method. This addition was made at the request of the late Earl STANHOPE; but it is proper to mention, that it does not appear that Dr. SIMSON ever intended to make any selection of such problems, with a view to publication. There are, no doubt, among his papers, a great number of geometrical problems; some of which are valuable, but almost all of them appear to have been taken up only from the suggestion of the moment, either from his reading, his correspondence, or from his own particular studies; but generally without any decisive marks of his having given a particular consideration of the best possible solutions. The few added in this place were selected just when the volume was nearly printed, and without any parti-

* It appears from Dr. SIMSON's papers, that he had been thinking of the method of limits as far back as 1736, and then proposed to extend this method to the first and some other Propositions of the *Principia*. No trace however remains of his having compleated this design.

cular object in view, except that examples might be given of the accurate determination of problems, about which the ancient Geometers were ſo very curious, and which has often been neglected by the moderns. I muſt obſerve, however, that the laſt problem, being a caſe of the *Tactions* of APOLLONIUS, will be mentioned more particularly afterwards.

SECTION IV.

Of Dr. SIMSON's *Unpubliſhed Papers and Correſpondence.*

I SHALL conclude the account of Dr. SIMSON's labours and inventions with a few obſervations on his unpubliſhed papers, and his correſpondence.* From theſe papers it does not appear that he ever ſeriouſly embarked in the reſtoration of any other of the treatiſes in the τόπος ἀναλυομένος, except thoſe already mentioned. In an early MS. volume, indeed, he gives an arrangement of the caſes of the Problem of the Tactions, and in other papers the ſolutions of moſt of them.†

* Wherever in this Memoir I have made uſe of Dr. SIMSON's unpubliſhed papers, the reference to them is particularly remarked, that any inadvertence or miſtake of mine might not be attributed to him.

† The Tactions of APOLLONIUS were firſt reſtored by VIETA, in a treatiſe, which he calls APOLLONIUS *Gallus*. Afterwards they were reſtored by various mathematicians, both geometrically and algebraically. A Treatiſe of the Tactions by J. GUGL. CAMERER was publiſhed at Gotha and Amſterdam in 1795, and is mentioned with commendation by MONTUCLA, (tom. iii. p. 14,) but it contains only an edition of VIETA's treatiſe, with notes and additions, and a curious hiſtory of the Problem. The hiſtory is intereſting, from the accounts which it contains of the labours of ſome foreign mathematicians upon this problem, which are little known in this country. He gives the preface and *Lemmata* of the Tactions in Greek, with ſome various readings of ſeveral manuſcripts of PAPPUS. Though VIETA's ſolutions are elegant, yet they are in ſeveral reſpects deficient. There is not a full diſtinction either of the caſes, or of the neceſſary determinations. No analyſis is given, and no attempt to reſtore the APOLLONIAN ſolutions by the uſe of the *Lemmata* in PAPPUS, which had been aſſumed in the work of APOLLONIUS. See CAMERER *de Tactionibus*, p. 4, and p. 12.

It would feem, however, that as there was little difficulty in this problem, it had not been fufficiently interefting to engage him in a compleat reftoration of it. The ufe of one of the *Lemmata*, (Prop. 117, Lib. vii. PAPPI,)‡ in refolving a cafe of the Tactions, did not for fome time occur to him; but having difcovered it, he deduced from it the elegant folution of that cafe which is printed in the pofthumous volume. I found among his papers the following date of this folution: "Feb. 9, "1734, mane, poft horam I^{mam} ante meridiem." So minute was he in fome of thefe notices; and in this cafe perhaps, at the moment, he felt a little fatisfaction from having overcome the only difficulty in reftoring the Tactions in the APOLLONIAN method.

PAPPUS, in his defcription of the Tactions, obferves that there are other problems refpecting Tactions, which were generally neglected by the editors; of whom, however, fome prefixed one of thefe problems to the firft of the two books of APOLLONIUS, as an eafy and proper introduction to the doctrine of Tactions.* PAPPUS gives the general defcription of this problem, from which Dr. SIMSON ftated the arrangement and folution of the feveral cafes; and he alfo propofed and refolved another problem of Tactions, viz. "of points, lines, and "circles, any two being given to defcribe a circle through "the points, or touching the ftraight lines or circles, of which

‡ See Note E. at the end.

* This Problem is "of points, ftraight lines, and circles, given in pofition any two, "to defcribe a circle given in magnitude, which may pafs through the given point "or points, and touch alfo (if poffible) the given lines or line." This problem was "alfo refolved by MARINUS GHETALDUS.

"the centre ſhall be in a ſtraight line given in poſition." But all theſe problems, though very eaſy, may be occaſionally uſeful, when a propoſed problem can be reduced to a caſe of any of them.

With reſpect to the *Inclinations* of APOLLONIUS, he ſeems to have beſtowed little attention; only it may be obſerved, that, after repeated unſucceſsful trials, he found (February 20, 1723) the ſolution of the problem about the Rhombus, by the Lemma of PAPPUS, (Prop. 70, lib. vii.) which APOLLONIUS no doubt had employed in his ſolution. The figure of this Lemma in COMMANDINE's PAPPUS, which is erroneous, was at the ſame time amended.* The *Inclinations* had been reſtored by MARINUS GHETALDUS; and ſubſequently to Dr. SIMSON's time, by Dr. HORSLEY, the late Biſhop of St. Aſaph, in 1772; and alſo by Mr. WALES, in 1779. I may further remark, that Dr. SIMSON made ſome uſeful corrections of another Lemma of the *Inclinations*, viz. Prop. 85, lib. vii. PAPPI. And in this Propoſition is an example of the deduction of a problem to a caſe of the *Sectio Determinata*, which is thence conſidered by PAPPUS as compleatly reſolved; and in the demonſtration a reference is made to that caſe, being *Epitagma* 2. Prob. 3. lib. i. *Sect. Determ.* In COMMANDINE's time the loſt treatiſe *De Sect. Determ.* had not been reſtored; and therefore, in his commentary on this Propoſition, he gives a ſolution of that caſe, as neceſſary to the ſolution of this lemmatic problem.

* See Note F. at the end, in which is given Dr. SIMSON's ſolution, and the amended figure of the Lemma. It may be obſerved, that an ingenious correction of the Lemma, and of its figure, as it is delineated in the Savil. MS. No. 3, is given by Dr. HORSLEY, in his *Reſtitution of the Inclinations*.

Among the Doctor's papers there remain alſo ſome fragments reſpecting *Loci ad ſuperficiem*, of which EUCLID compoſed two books, making a part of the τόπος ἀναλυομένος. PAPPUS, in his preface to his ſeventh book, gives no deſcription of theſe two books, nor any account of their contents, from which, as in other caſes, geometers might have been enabled to explain or reſtore them. There are, indeed, a few *Lemmata* at the end of his ſeventh book, ſaid there to belong to the *Loci ad ſuperficiem*, but the firſt, as it is enunciated, and the other four,*

* Theſe five *Lemmata* are in COMMANDINE's tranſlation, and ſome of the MSS. in a very imperfect ſtate, as will appear from the following ſhort account of them.

The enunciation of Lemma 1, (Prop. 235.) is a ſimple Conic *Locus*, but in a ſort of ſupplementary enunciation is an alluſion to a *Locus ad superficiem*, which in its preſent ſtate is unintelligible; but a conjecture about the meaning of it is propoſed by Dr. WALLIS, in the Savilian copy of COMMANDINE.

In fol. 300. a. COMMAND. 1588, the four laſt lines, though in the type of the commentary, is a part of the text, and the Greek, of which it is a verſion, is in the Savil. MS. No. 3. It is the enunciation of a well-known Conic *Locus*, but without any demonſtration; and at the end it is ſaid, "that it will be ſhewn by premiſing the "following *Locus*." This probably was Lem. 2, as the next Propoſition 236 (viz. the *Locus* referred to) is Lem. 3.

Prop. 236. Lem. 3. is alſo a well-known Conic *Locus*, conſiſting of three caſes, belong to the three ſections; but the firſt caſe of the parabola is alone demonſtrated in this Propoſition.

Prop. 237. Lem. 4. This is the enunciation of the two caſes omitted in the preceding Prop. 236; viz. thoſe of the ellipſe and hyperbola, and theſe are analyzed and demonſtrated.

Prop. 238. Lem. 5. This propoſition is only a repetition of what I ſuppoſe to have been Lem. 2, and is the ſame *Locus* to the three ſections as was there ſtated. PAPPUS demonſtrates only the firſt caſe of the parabola, by a reference to Prop. 236; but the other two caſes of the ellipſe and hyperbola are eaſily demonſtrated by a ſimilar reference to Prop. 237.

I may further remark that the *Locus ad hyperbolem*, in Prop. 237, is aſſumed by PAPPUS in the ſolution of a ſolid problem, Prop. 34, b. iv. without however quoting either Prop. 237 of b. vii. or APOLLONIUS. And no doubt it was at the time a well-known ſolid *Locus*.

are all folid *Loci*; and truly contain only two *Loci*, of which the one is preliminary to the other. What follows the enunciation of the firft (viz. Prop. 235) in its prefent mutilated ftate is unintelligible; it has, however, fome allufion to a *Locus ad fuperficiem*, but the other four have none whatever. It is indeed fomewhat fingular, that thefe Propofitions are introduced in this place. PAPPUS, in his preface, profeffes to give an account of the treatifes of the τόπος ἀναλυομένος, only fo far as the *Conics* of APOLLONIUS, in the arrangement affumed by him, which is not the order of time, excluding thereby (from defign, we may fairly fuppofe) the *Loci ad fuperficiem*, and feveral others. Perhaps thefe *Lemmata* may be a fragment of fome other work, as probably is the Propofition, called λῆμμα τοῦ ἀναλυομένου τόπου, fubjoined to them in feveral MSS. which has no obvious connection with any other Propofitions of that feventh book. It is poffible, however, that thefe five Propofitions may have been *Lemmata in Locos ad fuperficiem*; but being folid *Loci*, might have been confidered as a proper addition to the "*Lemmata in Conica* APOLLONII," to which they are fubjoined.

In the defcription of the *Loci Plani*, however, a fhort account is given of the *Loci ad fuperficiem*, and they are mentioned alfo in Prop. 28 and 29, lib. iv. PAPPI; which are problems refolved by means of *Loci ad fuperficiem*, and which Dr. SIMSON has corrected and rendered intelligible. Thefe are all the materials remaining for the inveftigation of this difficult and curious, though perhaps not very important, piece of ancient geometry.

Dr. SIMSON had begun very early to confider thefe *Loci*; for in 1721 I find fome equations for expreffing the relation of the co-ordinates, by means of three indeterminate quantities. It appears alfo that he had afterwards defigned to give a regular explanation of thefe *Loci*; for there remains a preface for fuch an effay, in which he propofes to explain the method of inveftigating fuch *Loci* by examples. From thefe he hopes that it would be eafy for one well acquainted with folid *Loci* to inveftigate and defcribe any *Loci ad fuperficiem* which can arife from Conic Sections varioufly fituated; ("*vario fitu difpofitis.*") In the beginning of the preface, where he remarks the very little known of thefe *Loci* among the moderns, he has a marginal correction, intimating that he had not, at the time of writing this preface, feen the treatife *Des Courbes a double Corbure*, which he afcribes (furely from inadvertence) to D. N.* But as there is no treatife of that period of this title, or indeed on this fubject, but that of M. CLAIRAUT, publifhed in 1730, there can be little doubt of that being the work alluded to by Dr. SIMSON.†

* M. NICOLE, in a Memoir of the Academy of Sciences, (1734) but fubfequent to the publication of CLAIRAUT's work, makes an allufion to the curves of double curvature. M. NICOLE was one of the cenfors of CLAIRAUT's treatife.

† The very flight notice of the ancient *Loci ad superficiem*, by DES CARTES, in his Geometry, (book ii.) was of no ufe for Dr. SIMSON's inveftigation. "At vero duabus "conditionibus deficientibus ad hujus puncti determinationem, Locus in quo illud "reperitur fuperficies eft," &c. This, to be fure, correfponds to an equation with three unknown quantities, but it is not purfued by DES CARTES. His commentator SCHOOTEN gives two examples, which may perhaps be included in the general defcription of the *Loci ad superficiem* by PAPPUS, in his account of *Loci Plani*; but the Data and the Locus are all in the fame plane, and they furnifh no illuftration of what Dr. SIMSON confiders as the object of the ancient treatife of EUCLID.——See CARTESII *Geomet.* Amft. 1683, p. 34; and SCHOOT. *Comment.* p. 228. See alfo SCHOOTENII *Sectiones Miscellaneæ*. Lugd. Bat. 1657, p. 494.

All theſe are comprehended in the following equation:

$$xx+\frac{a}{b}xy+ax+by+\frac{a}{c}yy+\frac{a}{d}yz+\frac{a}{e}xz+gz+\frac{b}{k}zz= \text{Dato.}$$

The ſigns being any how changed.*

In proceeding however to examples, he treats the ſubject geometrically, and without any reference to the equation. There are indeed ſeveral examples, but all of them, except a few of the ſimpleſt forms, are unfiniſhed; and though in the firſt there is a conſiderable progreſs, and an inveſtigation of the common ſection of the *Locus ad superficiem* ſought, when cut by ſeveral planes in different poſitions, yet he mentions ſeveral others to be aſcertained, but which are not further proſecuted. The Doctor was not eaſily diverted from an inveſtigation, merely from the difficulty of it; and we may therefore reaſonably infer, that he abandoned the ſubject, partly from its having been previouſly taken up by CLAIRAUT, and partly from conſidering it as a branch of ancient geometry, of inferior importance and utility.† The omiſſion of any deſcription of this and ſome other treatiſes of the τόπος ἀναλυομένος, in the preface to the ſeventh book, is a preſumption of a ſimilar eſtimation of the *Loci ad ſuperficiem* being entertained by PAPPUS.‡

* See Note G.

† See Note G. at the end, ſome of the eaſieſt examples.

‡ In vol. ii. p. 151, of FERMAT's works, there is a letter of his to ROBERVAL, in which, after ſome obſervations about *Loci*, and particularly reſpecting one of the ſecond book of APOLLONIUS, he adds, "*de Locis solidis et ad superficiem multa quoque* "*sunt detecta.*" He ſlightly alludes to the ſame ſubject in a letter to Sir KENELM DIGBY. WALLISII *Opera*, tom. ii. p. 859. Alſo in page 116 of ſecond vol. of FERMAT's works, in the preface to his Poriſms, after mentioning the reſtoration of ancient treatiſes of geometry by SNELLIUS, VIETA, and MARINUS GHETALDUS, he adds, "ſequebantur *Loci plani*, *Loci solidi*, et *Loci ad superficiem*, et huic quoque

Besides these fragments of geometrical speculations, there are a great variety of Problems, Theorems, Loci, Data, and Porisms, dispersed through the Doctor's papers, and particularly in the MS. volumes, which he called *Adversaria*, of which eighteen remain. These in general, as has already been observed, appear to have been analyzed or demonstrated, as the circumstances of his reading, conversation, or study, happened to suggest; yet they may be considered as a valuable repository of such Propositions, from which useful selections might be made, for promoting the study of geometry in the ancient method. They are deposited in the College library of Glasgow; and it is much to be wished, that some gentleman acquainted with the subject, and who has leisure and inclination for the task, would take the trouble of arranging a proper collection.*

It has been repeatedly remarked, that notwithstanding Dr. SIMSON's early and continued partiality for geometry as

" parti non ignoti nominis geometræ, succurrerant, eorumque opera manuscripta, " licet et adhuc inedita, latere non potuerint." And in another letter to Mr. ROBERVAL (p. 153) he observes, " J'en ay plus de cens Propositions tres belles et " particulierement des *Lieux solides* et *ad superficiem*, mais le loisir me manque, &c." No further account however of these restorations of *Loci ad superficiem* is to be found.

* There are in these volumes, also, a number of geometrical demonstrations, by the Doctor, of theorems, in the works of various eminent analysts, where they are treated either algebraically, or if geometrically, with inferior skill.—I may remark that most of the Propositions alluded to are dated; and on some particular occasions not only the day, but even the hour is noted. Sometimes also, when he had accomplished a solution or a demonstration, which pleased at the moment, some expression of his satisfaction is added. He began this practice very early, even before he was elected Professor of Mathematics, and it was continued till about fourteen months before his death. In the book of *Adversaria* Q. is the latest date now to be found, 11th August 1767; he being then in his eightieth year.

cultivated by the ancients, he did not entirely neglect the modern analysis, but became well acquainted with algebra, and with the common applications of it which were pursued in the early part of the last century.* He certainly did not approve of the prevailing use of algebra in geometrical investigations, which could be accomplished with more elegance, and not seldom with more ease, by the ancient method; but he made no scruple of acknowledging the superior power of algebra in complex geometrical inquiries, and more especially in the application of it to such as are physical.—It is probable however, from all the information which can now be obtained respecting his studies, that though he understood the modern analysis as delivered by Sir ISAAC NEWTON, yet he did not continue to devote much of his time to the study of it, and thence did not enter much into the profound analytical speculations of succeeding mathematicians.

It must be admitted, also, that his rigid notions respecting algebraical reasoning, particularly in geometry, had a tendency to limit investigations, by that method; and to discourage him from pursuing the methods of modern analysts, who felt no such difficulties as had occurred to him. As an example, however, of Dr. SIMSON's knowledge of algebra, I may mention his invention of some very simple and excellent serieses for finding the circumference of a circle, of which he appears to have given an intimation in 1722 to Dr. JURIN, then secretary of the Royal Society.—Dr. JURIN undertook to obtain the sentiments

* Dr. SIMSON often amused himself, especially in the latter part of his life, in resolving any geometrical problem which attracted his notice, by the algebraical method, though he rarely made any entry of these solutions in his *Adversaria*.

of his learned friends on these serieses; and therefore Dr. SIMSON, in a letter of Feb. 1, 1723, transmitted to him his paper on the subject,* containing the serieses, with two elementary Propositions from which they were derived. Though Dr. JURIN was not apprized that any thing of the kind had been done by others, yet by inquiry he found that Mr. MACHIN, near twenty years before, had communicated to the Royal Society some serieses of the same form, but had afterwards withdrawn them. Mr. MACHIN readily gave a copy of his serieses to be communicated to Dr. SIMSON, which Dr. JURIN sent to him immediately after; (March

* I annex a copy of the portion of this letter which relates to the serieses. The remainder of it respects the Porisms, and is taken notice of in another place. For the paper enclosed, see Note H. at the end.

"SIR,

"Friday last I had the favour of your very civil answer to the letter I lately gave "you the trouble of: some private affairs have kept me from transscribing the "enclosed trifle, else you should have had it sooner.—The series for finding the "circumference of the circle which are in Mr. SHERWIN's Book of Tables, being "deduced from tangents of arches which have a given ratio to the circumference, I "was desirous to try if the like might not in a general way be drawn from tangents, "whose arches are not in a given ratio to it; and found a great many easily flowed "from the two Propositions and their corollaries, which I have enclosed. The Pro-"positions, though obvious enough, I do not find in Mr. BRIGGS, or any of the authors "I have searched for them: the corollaries are long since known, if I mistake not. "Among the series I have set down, there is one which Mr. JONES, in his Synopsis, "says he had from Mr. MACHIN, and which is certainly one of the best that can be "found by this or any other method. Whether any or all of the rest have been taken "notice of by any body, I am wholly ignorant; but you may soon discover.

"I thankfully acknowledge the favour of your kind offer to obtain the sentiments "of some of your learned friends upon the enclosed paper, but I entreat you may "give yourself no more trouble about it, since I am ashamed to have given you "already so much, when the thing is scarce worth taking notice of, and which you "will easily see is not to be shewn to any but as such."

"Glasgow, Feb. 1, 1723."

1723). The extreme modeſty with which Dr. SIMSON mentions this ingenious inveſtigation to Dr. JURIN, is remarkable; for though from the preſent ſtate of ſcience, thoſe ſerieſes may have loſt their former importance, yet near a century ago they had the merit of novelty and utility, and had then occupied the attention of Mr. MACHIN and Dr. SIMSON. It may thence be ſatisfactory to ſome readers to ſee a fuller account of them, which is given in a note at the end.*

The Mathematicians of the end of the ſixteenth and beginning of the ſeventeenth centuries began to apply algebra to geometrical inquiries; and this method was much promoted and extended by DES CARTES. The uſe of negative and impoſſible roots of equations was alſo introduced into this analyſis; though by preceding algebraiſts they were at firſt not obſerved, and afterwards when obſerved were reckoned uſeleſs. While algebra, in its early and humble ſtate, was employed almoſt entirely about numbers, theſe roots were naturally neglected, and the language of the firſt writers on the ſcience was thence not encumbered with the metaphyſical difficulties which aroſe from the proceſſes and reaſonings of the followers of DES CARTES. Some of theſe were reaſonably conſidered by Dr. SIMSON as defective in that preciſion of definition, and ſtrictneſs of argument, which have ever been the boaſt of pure geometry, and to which he had been accuſtomed from his almoſt conſtant ſtudy of that branch of the ſcience. The Doctor, from this cauſe perhaps, conceived a prejudice againſt an application of algebra, which

* See Note H.

was accompanied with ſuch difficulties; and was thence led to treat of that ſcience after the manner of the early writers on it, with the limited definition and uſe of the negative ſign. Many detached obſervations on this ſubject remain among his papers;* and ſome ſhort eſſays alſo on cubic equations, in which he endeavours to explain them without admitting negative or impoſſible roots. In the preſent ſtate of that ſcience, it is unneceſſary to mention the detail of theſe ſpeculations; and it may be ſufficient to remark, that though the method of inveſtigation was different, the reſults were ſimilar to thoſe contained in the Eſſay on the Negative Sign by that learned promoter and patron of mathematical ſcience, Mr. BARON MASERES; of which, from the correſpondence of opinions, Dr. SIMSON ſpoke with much approbation. Dr. SIMSON's notions on this ſubject probably varied a little in the progreſs of life, by his confirmed partiality for the ancient analyſis, and an encreaſing prejudice againſt the algebraical. The elegance and ſatisfactory clearneſs of every ſtep in the geometrical method muſt, where it can be employed, be univerſally preferred to the almoſt mechanical proceſs of an algebraical calculation. His animadverſions, however, on the application of algebra to geometry, chiefly referred to thoſe caſes where it was not neceſſary, and in which the more excellent method of the ancients could be ſucceſsfully employed. In Plane Geometry, and in the Conic Sections, the ancient method is ſtill generally uſed, and by good judges is allowed to be the beſt for forming the

* See in Note K. at the end, ſome obſervations by Dr. SIMSON on this ſubject, in his letter to Mr. GEO. LEWIS SCOTT.

tafte, and exercifing the reafoning powers, of young perfons; and Dr. SIMSON has done effential fervice by his geometrical works, in exciting curiofity refpecting thefe branches, and facilitating the ftudy of them.

Dr. SIMSON has no where in his writings given any fpecific opinion of the proper and neceffary ufe of algebra in geometrical enquiries, and therefore we cannot judge with precifion of his views refpecting it. It may be obferved, however, that as the indefinite number of claffes of geometrical curves, in the prefent ftate of both the ancient and modern analyfis, can be conveniently treated of only by algebra, this application of algebra muft be admitted as neceffary; and the definition of the Conic Sections by equations may be regarded as an ufeful introduction to the definition of fuperior curves on that principle. Dr. SIMSON feems, in the early part of his life at leaft, in fome meafure to have admitted this; for in a great number of folid *Loci* among his papers, though demonftrated geometrically, the fuitable equation is added. Probably, however, he never fet about taking an accurate and comprehenfive view of the fubject in all its bearings; and having made his early choice from tafte, the ancient analyfis continued through life to be the favourite object of his ftudy.

Confidering the unbounded extent of mathematical fcience, and the variety of branches into which it may, and has been ufually diftributed; it may be for the advantage of the moft fuccefsful progrefs of the whole, that individual mathematicians fhould apply themfelves chiefly to particular divifions of it. Though there be a clofe connection among

these several branches, yet a moderate knowledge of the general system may be sufficient for the successful cultivation of any one portion of it, to which the attention of a mathematician may happen to be directed. Had Dr. SIMSON applied his ardent mind to the modern analysis, most probably the results of his investigations would have been important; but his not having devoted much of his attention to the study of it, was certainly favourable to his progress in the laborious researches which he undertook respecting the ancient geometry. It will be readily admitted also, that if some of the great modern analysts happen not to have studied the ancient geometry, as may be presumed to be the case, they were not thereby materially obstructed in their profound investigations, by which human knowledge has been so much enlarged; though, perhaps, by such previous study, their works might in some instances have been improved in simplicity and perspicuity.

It appears from the Doctor's letters to the late Earl STANHOPE,* that his Lordship had recommended to him to attempt the demonstration of some of the more important modern geometrical discoveries by the strict and more elegant method of the ancients. This was in 1751, and though he states it to be very much his own wish to make the trial, he considers it to be beyond his power, from the decline of his faculties. The tract on *Logarithms*, and the fragment *De Limitibus*, seem to be his only attempts of that kind; but he proposes to recommend it to his scholars, Dr. STEWART, Dr. MOOR, and Mr. WILLIAMSON. There are indeed among his papers a few imperfect sketches (apparently of an old date) of attempts to

* See Note I. at the end.

demonſtrate geometrically the celebrated theorem of the Circle invented by Mr. COTES. He does not ſeem to have beſtowed much time to this ſubject; but I have heard him mention that his plan was to demonſtrate firſt the more eaſy caſes, and then to attempt to frame a general method of deducing any caſe of this Propoſition from the next preceding and more ſimple caſe, from which a compleat demonſtration of the theorem in its full extent might be derived. But in this he was unſucceſsful, and he regretted the diſappointment. But as neither Dr. SIMSON, nor any other geometer, ſo far as is known, have ever accompliſhed a geometrical demonſtration of this theorem, we may conſider the inveſtigation of Mr. COTES as an important example of the ſuperior power of the modern analyſis, in the diſcovery and demonſtration of a Propoſition, very general and difficult indeed, but ſtrictly geometrical; and though it be commonly expreſſed algebraically, yet may, by means of compound ratios, be enunciated in the pure language of geometry.

Long before any ſymptom of decline in his health appeared, he ſeems to have felt ſome remarkable decay of his memory and attention. This was not obſerved by his friends, either in converſation or in correſpondence; but his complaints of it were ſo particular and ſerious, that no doubt can be entertained of the fact, nor of the influence which it had in diverting his mind from ſome of the more ſevere inveſtigations which he had formerly wiſhed to purſue.*

In the preface to the Doctor's *Conic Sections*, when recommending the geometrical method of treating the ſubject, in

* See Note K. at the end.

preference to the algebraical, he has this remarkable ſentence: "In quibus autem differat analyſis geometrica ab eâ quæ "calculo inſtituitur algebraica, atque ubi hæc aut illa ſunt "uſurpanda, atque quæ ſint in mathematicis utriuſque partes "propriæ, alias differendum." From this obſervation it muſt be acknowledged, there naturally ariſes a preſumption, which has been generally entertained, that the Doctor had conſidered the ſubject, and probably written ſomething concerning it: and the opinion of ſo diſtinguiſhed a mathematician on a point of ſome difficulty, and of which he was well qualified to judge, would have been particularly intereſting. But among his papers no trace of ſuch an attempt is to be found. Profeſſor Robison, in his Life of Dr. Simson already mentioned, ſeems to think that a diſſertation on *Loci*, which he had ſeen when attending the Doctor's Lectures in the College of Glaſgow, might have been the tract to which the preceding quotation referred.* Of this, however, notwithſtanding the weight of Mr. Robison's opinion, a reaſonable doubt may be entertained, conſidering the extent of the diſcuſſion implied in that quotation, which muſt have gone far beyond what ſuch a diſſertation as Mr. Robison mentions, could be ſuppoſed to contain. But Dr. Simson's own teſtimony, in his correſpondence with Mr. Scott reſpecting this very ſentence, is concluſive, and puts an end to all ſpeculation regarding it, as he therein explicitly declares, that on the ſubject of it "he had no "papers, and never wrote any thing on that matter." Fortunately, a conſiderable part of this correſpondence is preſerved; and the moſt important portion of it is placed in a note at the

* No trace of this diſſertation on *Loci* remained among his papers.

end.* The letters both of Mr. SCOTT and Dr. SIMSON will be read with intereſt: they contain ſome valuable obſervations on the relative merits of the ancient analyſis and of the algebraical, in the ſolution of geometrical problems. The ſubject indeed is not exhauſted; and as the correſpondence took place from occaſional circumſtances, the diſcuſſion, though entertaining and inſtructive, is of courſe limited very much to the points which thus accidentally aroſe. It touches little on the peculiar powers of the modern analyſis, and is confined chiefly to the compariſon of the two methods in caſes where either may be employed.†

* See Note K.

† For ſome remarks on this correſpondence, ſee the latter part of Note K.

SECTION V.

Sketch of Dr. SIMSON'*s Character.*

IT was formerly obferved that Dr. SIMSON'S mathematical labours and inventions, of which fome account has been given in this memoir, were the moft interefting occurrences of his life, and moft illuftrative of his capacity and genius.

In attempting, however, to give a brief outline of the Doctor's character, befides the talents difplayed in his geometrical works, which claim the firft confideration, it may be expected that fome obfervation alfo fhould be made of the well-known difcriminative features of his mind, which, though not important in themfelves, appear to have had an influence on its greater qualities. Thefe peculiar features are ftill affectionately remembered by his pupils, friends, and acquaintances who furvive; and, from the eminence of his genius, they may be regarded by others as objects of a natural and liberal curiofity.

Dr. SIMSON was originally poffeffed of great intellectual powers, an accurate and diftinguifhing underftanding, an inventive genius, and a retentive memory: and thefe powers, being excited by an ardent curiofity, produced a fingular capacity for inveftigating the truths of mathematical fcience. By fuch talents, with a correct tafte, formed by the ftudy of

the Greek Geometers, he was alfo peculiarly qualified for communicating his knowledge, both in his lectures and in his writings, with perfpicuity and elegance. He was at the fame time modeft and unaffuming; and though not indifferent to literary fame, he was cautious and even referved in bringing forward his own difcoveries, but always ready to do juftice to the merits and inventions of others. Though his powers of inveftigation in the early part of life were admirable, yet before there appeared any decline of his health, he felt ftrong impreffions of the decay both of his memory and other faculties; occafioned probably by the continued exertion of his mind in thofe fevere ftudies, which for a number of years he purfued with unremitting ardour.

Befides his mathematical attainments, from his liberal education he acquired a confiderable knowledge of other fciences, which he preferved through life, by occafional reading, and in fome degree alfo, by his conftant intercourfe with fo many literary men in his College.† He was efteemed a good claffical fcholar, and though the fimplicity of geometrical demonftration does not admit of much variety

† Dr. SIMSON in the early part of his life ftudied Botany, in which he made great progrefs, and it became a fource of amufement to him in his walks. In the year 1746, when the Univerfity of St. Andrew's wifhed to confer on him the degree of Doctor, as he was a layman, a degree of medicine was propofed from the circumftance juft mentioned, though he had no other pretenfion to diftinction in that fcience.

It may be mentioned alfo in this place, that a few years after he became a Profeffor, he compofed a tract on the School Logic, with rigorous demonftrations of its rules. Some fketches of it remain, which ftrongly mark the accuracy of his reafoning powers to whatever fubject they were directed.

of ſtile, yet in his works a good taſte in that point may be diſtinguiſhed. In his Latin prefaces alſo, in which there is ſome hiſtory and diſcuſſion, the purity of language has been generally approved. It is to be regretted indeed that he had not had an opportunitity of employing, in early life, his mathematical and Greek learning, in giving an edition of Pappus in the original language.‡

Dr. Simson never was married; and the uniform regularity of a long life, ſpent within the walls of his College, naturally produced fixed and peculiar habits, which however, with the ſincerity of his manners, were unoffending, and became even intereſting to thoſe with whom he lived. The ſtrictneſs of theſe habits, which indeed pervaded all his occupations, probably had an influence alſo on the direction and ſucceſs of ſome of his ſcientific purſuits. His hours of ſtudy, of amuſement, and of exerciſe, were all regulated with uniform preciſion. The walks even in the ſquares or garden of the College were all meaſured by his ſteps, and he took his exerciſe by the hundreds of paces, according to his time or inclination.

‡ Dr. Simson's learned diſcuſſion concerning ſome Greek terms in the account of the Poriſms by Pappus has already been mentioned, and a copy of it is inſerted in Note C. I may mention here alſo another example of his critical knowledge. In the dialogue of Plato, Meno, there is a paſſage, of which, even in the beſt editions, ſome ſentences are ſomewhat obſcure, and probably corrupted. It relates to an ingenious ſpeculation, of all knowledge being only reminiſcence; and as an illuſtration, an uninſtructed youth is led, merely by an examination, to aſſent to the truth of a geometrical propoſition, (an eaſy caſe of the 47. 1. Elem. Eucl.) This had attracted Dr. Simson's notice; and he found that ſome natural and eaſy emendations, with the addition of a diagram, which, in the dialogue, is plainly ſuppoſed to be exhibited to the youth, rendered the argument intelligible and correct. He had drawn up a detailed ſtatement of the paſſage, and the neceſſary corrections which, in his own time I ſaw in the hands of one of his colleagues, who has been dead many years: but no trace of it is now to be found among the Doctor's papers, or any where elſe.

It has been repeatedly mentioned, that an ardent curiofity was an important feature of his character. It contributed effentially to his fuccefs in his mathematical inveftigations, and it difplayed itfelf in the fmall and even trifling occurrences of common life. Almoft every object and occurrence excited it, and fuggefted fome problem which he was impatient to refolve. This difpofition, when oppofed, as it often neceffarily was, to his natural modefty, and to the formal civility of his manners, occafionally produced an embarraffment, which was amufing to his friends, and fometimes a little diftreffing to himfelf.

The Doctor, in his difpofition, was both cheerful and focial; and his converfation, when he was at eafe among his friends, was animated and various, enriched with much anecdote, efpecially of the literary kind, but always unaffected. It was enlivened alfo by a certain degree of natural humour; and even the flight fits of abfence to which in company he was occafionally liable, contributed to the entertainment of his friends, without diminifhing their affection and refpect, which his excellent qualities were calculated to infpire. One evening in the week he devoted to a club, chiefly of his own felection, which met in a tavern near the College. The firft part of the evening was employed in playing the game of whift, of which he was particularly fond; but though he took no fmall trouble in eftimating chances, it was remarked that he was often unfuccefsful. The reft of the evening was fpent in cheerful converfation, and as he had fome tafte for mufic, he did not fcruple to amufe his party with a fong; and it is faid that he was rather fond of finging fome Greek odes, to which modern mufic had been adapted. On Saturdays he ufually dined in the village of

Anderſton, then about a mile diſtant from Glaſgow, with ſome of the members of his regular club, and with a variety of other reſpectable viſitors, who wiſhed to cultivate the acquaintance, and enjoy the ſociety, of ſo eminent a perſon. In the progreſs of time, from his age and character, it became the wiſh of his company that every thing in theſe meetings ſhould be directed by him; and though his authority, growing with his years, was ſomewhat abſolute, yet the good-humour with which it was adminiſtered, rendered it pleaſing to every body. He had his own chair and place at table; he gave inſtructions about the entertainment, regulated the time of breaking up, and adjuſted the expenſe. Theſe parties, in the years of his ſevere ſtudy, were a deſirable and uſeful relaxation to his mind, and they continued to amuſe him till within a few months of his death.

Strict integrity and private worth, with correſponding purity of morals, gave the higheſt value to a character, which, from other qualities and attainments, was much reſpected and eſteemed. Upon all occaſions, even in the gayeſt hours of ſocial intercourſe, the Doctor maintained a conſtant attention to propriety. He had ſerious and juſt impreſſions of religion, which appear occaſionally in his papers, and may be traced even in the midſt of ſome of his mathematical inveſtigations;* but he was uniformly reſerved in expreſſing particular opinions on the ſubject of religion; and from his ſentiments of decorum,

* As an example, the following date of the ſolution of a problem, which happened to be his birth-day, may be quoted, as it ſtands in one of his MS. volumes:

" 14 Octobris 1764
" 14 Octobris 1687
" 77

" DEO Optimo Maximo, Benigniſſimo Servatori, ſit laus et gloria."

he never introduced religion as a ſubject of converſation in mixed ſociety, and all attempts to do ſo in his clubs were checked with gravity and deciſion.

Dr. SIMSON in his perſon was tall and erect, and his countenance, which was handſome, conveyed a pleaſing expreſſion of the ſuperior character of his mind.* His manner had always ſomewhat of the faſhion which prevailed in the early part of his life, but was uncommonly graceful. He was ſeriouſly indiſpoſed only for a few weeks before his death, and through a very long life had enjoyed an uninterrupted ſtate of good health. He died on the firſt of October 1768, when his eighty-firſt year was almoſt compleated; having bequeathed his ſmall paternal eſtate in Ayrſhire to the eldeſt ſon of his next brother, in whoſe family it ſtill remains.†

* In the College hall of Glaſgow is a portrait of Dr. SIMSON, which, though painted when he was in the vigour of life, yet expreſſes a likeneſs to his countenance and figure as they appeared in advanced age. On the title-page of this Memoir is placed a ſmall engraving from this picture, with an inſcription written by the late Dr. MOOR, Greek Profeſſor at Glaſgow, happily expreſſing Dr. SIMSON's diſapprobation of the modern uſe of algebra in geometry, and his ſingular merit and ſucceſs in reſtoring the ancient analyſis. See the Edinburgh Tranſactions, vol. i. p. 75. Hiſtory.

† Dr. SIMSON, from his conſtant reſidence in the College, and his engagement in ſtudy, gave little attention to the improvement of his eſtate, which deſcended to him encumbered with debts, and which became much more productive to his heir than it had been to himſelf. He was at the ſame time well acquainted with buſineſs, having been for a great number of years Clerk to the Faculty, and in that character he had the chief management of the property and other affairs of the College, which he conducted with his habitual regularity and accuracy. In his manner of living he was ſimple and unexpenſive, except in collecting a valuable mathematical library, which he bequeathed to his College. But from an income, ample with reſpect to his habits, he made no accumulation; having enjoyed the ſatisfaction of advancing young relations, in his own time, by many acts of generoſity.

NOTES.

NOTES.

NOTE A. p. 28.

FROM APOLLONIUS and his commentators to MYDORGIUS and others of the seventeenth century, Dr. SIMSON considers this branch of geometry as stationary for that long period. It has been properly observed however by different authors, that the points, in subsequent times called *Foci*, were ascertained by APPOLLONIUS only in the ellipse and hyberbola, but not in the parabola. The Rev. Dr. ROBERTSON, Savilian Professor of Astronomy at Oxford, in his learned History of Conic Sections, annexed to his Treatise on that subject,* after mentioning this observation, adds, that in two small tracts, now very rare, edited by GOGAVA, Louv. 1548, the first mention is made, so far as he knew, of the focus of the parabola.† GOGAVA considers the second tract (*De Speculo Ustorio*) as of Arabic origin, and very ancient; and also that VITELLO had borrowed from it. VITELLO flourished about 1470,

* *Sectionum Conicarum, Libri VII.* &c. Oxon. 1792, pp. 340, 362.

† This publication appears to be a first edition, though other dates are given to it by writers who quote it; and it has a Preface by GEMMA FRISIUS. The second of these two small tracts seems to have been generally known before the edition by GOGAVA. ROGER BACON quotes it; and VERNERUS, in his tract *De Cubo Duplic.* Nurem. 1522, append. 12mo. plainly alludes to it. MAUROLYCUS also, in a miscellaneous volume, (Messanæ, 1558,) states

near a century before GOGAVA; and in his *Optics*, prop. 42, 43, book ix. (page 400, edit. Resneri,) the focus of the parabola, and the application of it in forming a concave parabolic mirror are diſtinctly deſcribed, but without ſtating the ſource from which he derived them.

It is plain, however, that the point, afterwards called the focus of the parabola, was uſed by PAPPUS, though not formally diſtinguiſhed or named, in prop. 238, lib. vii. (fol. 303. b. edit. 1588,) which is a *Locus* to the parabola. But he does not mention the property, eaſily flowing from that which he aſſumes, on which is founded the conſtruction of the parabolic reflecting mirror; and it is not improbable that this laſt property had not been obſerved in the time of PAPPUS, and that it was diſcovered long after by the Arabian opticians. This Prop. of PAPPUS muſt have been well known to Dr. SIMSON, though he appears not to have conſidered it as requiring notice in the ſhort hiſtory of Conics in his preface.

It may be convenient in this place to make an obſervation on another portion of the preface to Dr. SIMSON's *Conics*, viz. the paragraph beginning, "*Propositionum jamdudum, &c.*" (p. vi. Præfat. prim. et ſec. edit.) and ending "*contulerint.*"

This paragraph was added in the firſt edition, (1735,) in conſequence of an objection made by Mr. MAC LAURIN to a Propoſition of the Doctor's, as not containing a ſufficient acknowledgment of a communication by Mr. M. on the ſubject of it. Dr. SIMSON, in a letter to a common friend not named, which remains among his papers, ſtates this circumſtance. He ſays, "I "have not time to write about the Propoſition Mr. MAC LAURIN objects againſt. "To take away all controverſy in this matter, I ſend you an addition to the "preface;" viz. the above-mentioned paragraph. After ſome farther explanation of his anxiety to do juſtice to Mr. MAC LAURIN he ſays, "you may "ſhew him this addition to the preface, before printing it."—The explanation

the nature of this tract, obſerving, that it had been erroneouſly attributed to ARCHIMEDES, and it is in an abſtract of the works of ARCHIMEDES that he mentions it. He conſiders it, however, to be the work of PTOLEMY, without giving any authority. See the above mentioned volume of MAUROLYCUS, fol. 72. It was printed indeed ſome years after GOGAVA's publication; but probably MAUROLYCUS had not ſeen theſe ſmall tracts, as he makes no mention of them, or of the editor GOGAVA. The firſt tract, *De Parabola*, is ſuppoſed to be of the time of ROGER BACON, about 1270; but the ſecond, *De Speculo Uſtorio*, is much more ancient, being quoted by ROGER BACON.

feems to have been quite fatisfactory, for in the following year, 1736, the mathematical correfpondence between Dr. SIMSON and Mr. MAC-LAURIN continued on its ufual friendly footing.

Among Dr. SIMSON's papers there is an account of his *Conics*, drawn up foon after the publication, in the form of a letter, and in the name of fome friend of the Doctor's, apparently with the defign of being tranfmitted to a diftance, and perhaps for publication.* It is a fort of enlargement of the preface, giving a more particular detail of the author's views in compofing the treatife, and of the improvements which he had made in it. The following paffage in it gives a more particular account of the communication to Mr. MAC LAURIN. "The Scholium in p. 198, (firft edit.) and the Prop. it flows from, contain a "very general and ufeful property of the Sections, which the author difcovered "upon this occafion. When Mr. MAC LAURIN, the learned profeffor of "mathematics in the univerfity of Edinburgh, was going to France in the year "1723, Mr. SIMSON communicated to him a *Locus* of PAPPUS Alexand. "which he had reftored, viz. *Si a tribus punctis datis in recta linea ducan-* "*tur tres rectæ lineæ, et ipsarum intersectiones duæ tangant rectas positione* "*datas, tanget reliqua intersectio rectam lineam positione datam*: which was "afterwards printed in the Philofophical Tranfactions. When Mr. MAC LAURIN "had come home from France, he told Mr. SIMSON that he found, when the "three given points were not in a ftraight line, all the reft remaining as in "the former cafe, that the *Locus* defcribed would be a conic fection, and "that by help of this *Locus* he could eafily defcribe a conic fection through "five given points: This *Locus*, and method of defcription, he gave Mr. "SIMSON without any demonftration, which he faid he had made by help of a "*Lemma* in Sir ISAAC NEWTON's *Principles*; and it was in fearching for "the demonftration of them that Mr. SIMSON found out the Propofitions in "the Scholium, and thofe they flow from, which he did in the fame method

* Since writing this account, Mr. FRYER of Briftol, who is engaged in a hiftory of mathematics, pointed out to me a letter from the celebrated Profeffor HUTCHESON of Glafgow, on Dr. SIMSON's *Conic Sections*, which he had found in the *Bibliotheque Raisonée* for April, May, and June 1735; and which, on comparifon, I afcertained to be the very letter here alluded to. In that journal it is tranflated into French, and Profeffor HUTCHESON is mentioned as an old friend of M. SMITH, one of the publifhers, at Amfterdam. The letter is dated Glafgow, 28 Feb. 173$\frac{4}{5}$.

"they are now printed in his book." The copy of this letter, found among Dr. SIMSON's papers, is without any signature, but it is in the Doctor's handwriting.*

In the note * page 29, the proper reference is to BOSCOVICH *Sect. Con. Elementa.* art. 275.

NOTE B. p. 43.

To the examples mentioned by Dr. SIMSON, of misapprehensions entertained by eminent mathematicians respecting Porisms, may be added that of CASTILLON. In his commentary on the *Arithmetica Universalis*, (Amst. 1761,) p. 216, vol. 1, after quoting a passage from MARINUS's preface to the Data, he adds, "Hæc observatio mihi viam aperuit ad restituenda EUCLIDIS Porismata;" and afterwards, page 268, when speaking of the ancient problem of the Tactions, he adds, "omnia soluta habeo, et me editurum spero propediem cum Porismatis "EUCLIDIS restitutis.' The same author, in his commentary on the appendix, vol. ii. page 264, shews more particularly his misapprehension of the nature of Porisms, by supposing them to be the constructions of EUCLID's data; and then he adds, "ad hunc (sc. librum Porismatum) non pauca colligere "cæperam, sed cum audivissem virum doctissimum ROBERTUM SIMSONUM, "rem perfecisse, et sua scripta reliquisse, ab incepto destiti, sperans et rogans ut "tanti viri cogitationes in publicam lucem emittantur." Dr. SIMSON had got this book a few years before his death, and was much amused with these observations of the author. It is very surprising that so many respectable mathematicians deceived themselves on the subject of Porisms, especially those who were apprized of the failure of Dr. HALLEY, who candidly acknowledges it, but who, of all who attempted the investigation of them before Dr. SIMSON, seemed to be, from his talents and his knowledge, the most likely to succeed.

* In this letter also is given a solution of the problem, "To describe a Conic Section "which shall pass through two given points, and touch three straight lines given in position." —This he easily deduces from Prop. 12. b. v. (1st ed.) by means of the before-mentioned *Locus*, published in the Phil. Transactions for 1723, p. 330, without the transformations employed in Prop. 25 and 26 of NEWTON's *Princip.* lib. 1.

Note C. p. 22.

" I ſubjoin the remainder of Dr. Simson's letter to Dr. Jurin, of which the firſt part is in the text at page 22. " At his (Dr. Halley's) deſire I have " conſidered that paſſage as narrowly as I could, and ſhall be very glad to have " his opinion concerning the following conjectures; ſince, both with reſpect " to the language and the matter, I eſteem him the moſt ſkilful judge.

" The firſt thing to be conſidered is, that the figures Pappus ſpeaks of in " this Propoſition (for I ſuppoſe σχήματος, or ſome ſuch word, ought to be under- " ſtood to be joined with ὑπτία &c.) are ſuch as are made by four ſtraight lines, " either all of them interſecting one another, or two of them, at the moſt, " parallels: (the caſe of the parallelogram, becauſe of its plainneſs, being " omitted.) I mean, Pappus ſpeaks of the complex figures made by the " actual interſections of all the lines, and not of ſimple Trapezia, whoſe ſides, " if produced, would meet: this is plain from his ſuppoſing three points of " interſection in their ſides; ' Si dentur, in earum una, puncta tria.'

" Of the three kinds of theſe figures he mentions, that which he calls " παράλληλον is eaſily, and I think certainly, determined, from ſome places in " his Lemmas, to be the figure made by two parallel lines meeting two others " which are not; ſo that either of theſe two figures A or B is the παράλληλον.*

" The places which verify this are firſt in Prop. cxxxvi. at the letter C in " page 247† (of the edition at Ven. 1589,) Commandine has it, ' ita KH " ad HL hoc eſt in lineis parallelis GH ad HM.' where the Greek has " τουτ' ἔστιν ἐν παραλλήλῳ &c. and the figure GHMLK‡ is plainly ſuch as has been " deſcribed above. Again, in Prop. cxxxv. fol. 246, there occurs, " ut FM " ad DH, ita in lineis parallelis FK ad KH;' which I doubt not will be in " the Greek MSS. ἐν παραλλήλῳ, and the figure is the ſame as in the laſt.

" The ſame way of ſpeaking is twice uſed in the firſt demonſtration of the " firſt Lemma, fol. 238.§ And all the figures to which theſe two paſſages refer " are of the other kind, (viz. fig. B.) except one. And from this it appears " that παράλληλον is promiſcuouſly applied to both.

* Fig. A and B. † This ſhould be fol. 246. b. ‡ Fig. 1.

§ It appears that the MS. uſed by Commandine had ἐν παραλλήλῳ in this prop. (127.) See his Note D.

"It is also plain, that the passage in fol. 242, at letter B, is not corrupted, "as COMMANDINE suspects; ἔστι καὶ γὰρ ἡ θκ πρὸς τὴν θη ἐν παραλλήλῳ, which he "should have rendered into Latin thus, 'ut autem AF ad FG, ita LH ad 'HM, etenim in eadem ratione (sc. cum utrisque illis rationibus) est HK ad 'HG ἐν παραλλήλῳ, viz. est AF ad FG ut HK ad HG ἐν παραλλήλῳ AFGHK," 'and LH ad HM ut HK ad HG, ἐν παραλλήλῳ LHKGM.'

"Having determined the figure which PAPPUS calls παράλληλον, the rest, viz. "those where none of the lines are parallel, must belong to other two kinds, "ὕπτιον and παρύπτιον. The first of which words usually signifies *supinus, re-"supinus, restrorsum vergens*, and is applied by ARISTOTLE to the recurved "horns of Oxen. Now all the figures whatsoever that can be made by the "intersecting of four straight lines none of which are parallels, are with "respect to their form, of one kind; and have every one a reversed or retroflexed "angle, which will agree very well with the signification of ὕπτιος *restrorsum* "*vergens*. But, then, if every one of these figures be ὕπτιον, what comes of "the remaining branch of the division παρύπτιον? This is a difficulty cannot be "taken away, if we suppose PAPPUS to speak of the form of these figures "absolutely considered; but may be removed, if it be found he has regard "to the twofold manner in which they are formed, by two lines drawn "intersecting two other lines, and so as to give a just reason of using these "terms ὕπτιον and παρυπτιον respectively.

"I shall first explain the two ways after which these figures are formed, so "as to give rise to the names PAPPUS uses, and then shew what ground there "is to think he considered them thus, and not absolutely with respect to their "form.

"FIG. 2. Let two lines AB, AC, be drawn, intersecting two others DB, DC; "then if the angles BAC, BDC, made by these lines, lie one over another, *i. e.* "ὑπτίως; or which is the same thing, if one of them, BDC, be the reversed "angle of the figure; then such lines may be said to make the ὕπτιον σχῆμα.

"FIG. 3. But when the two lines AB, AC, are drawn intersecting two "others DB, DE, so that the two angles BAC, BDE, made by these lines, "do not lie below one another, but one of them is upon one side and the "other upon the other hand of the ὑπτία γωνία AFD, which they form by their "two nearest sides AC, DE, and so lie beside (παρὰ juxta) the reversed angle, "as also at one another's sides; then these pairs of lines may be said, with "respect to the angles they make, to form the παρύπτιον σχῆμα.

" And that PAPPUS had regard to these two different ways in which one pair " of lines meet another pair, I think will appear plain enough by comparing " Prop. cxxxvi. with Prop. cxlii.; in each of which he considers the figures we " are now speaking of, as formed by two lines intersecting two others; and these " figures are not only, absolutely considered, exactly the same, but the things " demonstrated are also the same, viz. the converse of Prop. cxxix.; nor is there " any thing to give occasion to the splitting it into two cases, other than the " consideration of the different ways the one pair of lines meets the other pair; " viz. in Prop. cxxxvi. the lines DH, HE, meet the lines BAE, DAG, ὑπτίως, " or so as to form the ὕπτιον σχῆμα. And in Prop. cxlii. the lines BD, DE, meet " the lines AB, AC, παρυπτίως, or so as to form the παρύπτιον σχῆμα with respect " to their angles.

" Some, perhaps, may incline rather to give παρὰ the negative or contrary " force, when joined with ὕπτιος; because the angles made by the pairs of " lines which form the figure, are the internal opposites of two triangles, of " which the reversed angle is the common external. I shall leave this to others " to determine, and shall only say, I do not remember παρὰ is used in this sense " in any mathematical term; for in EUCLID's Data I think it is quite amiss " to translate it *contra*."

He adds a Porism without demonstration, which it is unnecessary to insert, and concludes, with an apology for writing in haste,

" I am, Sir, your much obliged and
" affectionate Servant,

" GLASGOW, Jan. 10, 172½. ROBERT SIMSON."

NOTE [2 C.] p. 49.

On the subject of Porisms, Dr. SIMSON was certainly reserved; and this probably in part arose from a natural desire of publishing his discovery of them in a compleat form, without the risk of his being anticipated, which from partial communications was not unreasonably to be apprehended. Candour was a striking point of his own character; and we may observe in his works the precision with which he remarks the inventions, the im-

provements, and even the ſmalleſt hints contributed by his friends;* and perhaps he felt ſome diſſatisfaction from not having always met with the ſame liberality. Some inſtances were well known, and there were reports of others; but though they may account for his reſerve, it is neither of importance to his reputation, nor in any reſpect expedient, to inveſtigate or to record them His reſerve on the ſubject of Poriſms I had frequent opportunities of remarking, though favoured with much familiar converſation with him on mathematical ſubjects. From PAPPUS, and other writers, by whom Poriſms were mentioned, and alſo from ſome very diſtant alluſions from Dr. SIMSON, I certainly had got a general, but as I afterwards learnt, an imperfect notion of Poriſms. I conceived them to be entirely geometrical, and that they were a claſs of Propoſitions in which certain points or lines were to be found, which might have a *general* property expreſſed in the enunciation. I occaſionally ſubmitted to Dr. SIMSON ſome Propoſitions which I conſidered to be of that claſs; but without admitting or denying them to be Poriſms, with ſome pleaſantry he ſaid they were Propoſitions. It appears from his *Posthumous Works*, p. 459, that, with his uſual accuracy, a ſimple Propoſition of that kind (which I had ſtated to him as a problem) had been entered in the fair MS. of the Poriſms from which the treatiſe was printed.

NOTE D. p. 52.

As every notice of the Poriſms in former times is intereſting, I ſhall inſert ſome extracts from PROCLUS reſpecting them, in his Commentary on the firſt book of EUCLID's *Elements*. I quote from HERVAGIUS's edition, the only printed one, though very erroneous, as is obſerved by BAROCIUS in his tranſlation, who ſays it is "dilaniatum potius quam impreſſum." But as BAROCIUS had acceſs to ſeveral MSS. from which he choſe the beſt readings, his tranſlation, where it differs from the printed Greek, is a better authority; I therefore annex the tranſlation, both it and the original being ſcarce.

The firſt mention of the Poriſms is in page 58 of HERVAGIUS's edition, and in page 121 of BAROCIUS, viz. in the Commentary on the firſt Prop. of the *Elements*, where he profeſſes to give only a brief account of ſeveral mathe-

* See SIMSON's *Conics*, 2d edit. præfat. p. vi. and Prop. 19, lib. 5, at the end. See alſo *Loci Plani*, p. 223.

matical terms, such as Lemma, Porism, Case, &c.* The notice of the Porism is as follows:

"Τὸ δὲ πόρισμα λέγεται μὲν καὶ ἐπὶ προβλημάτων τινῶν οἷον* τὰ Εὐκλείδῃ γεγραμμένα, πορίσματα λέγεται δὲ ἰδίως ὅταν ἐκ τῶν ἀποδεδειγμένων ἄλλο τι συναφανῇ θεώρημα μὴ προθεμένων ἡμῶν ὃ καὶ διὰ τοῦτο πόρισμα κεκλήκασι ὥσπερ τι κέρδος ὂν τῆς ἐπιστημονικῆς ἀποδείξεως πάρεργον."

* οἷα.

"Corollarium vero, dicitur quidem et de quibusdam problematibus ut corollaria, quæ Euclidi ascripta sunt. Dicitur autem proprie corollarium, cum ex iis quæ demonstrata sunt quoddam aliud theorema apparuerit, nobis minime proponentibus, quod etiam propterea corollarium vocarunt, tanquam lucrum quoddam, quod sit præter gignentis scientiam demonstrationis propositum."

This very short description is unsatisfactory, but a subsequent passage (Proclus, Hervagii, p. 80. Baroc. p. 173.) is more particular and intelligible:

"Ἕν τι τῶν γεωμετρικῶν ἐστὶν ὀνομάτων τὸ πόρισμα. τοῦτο δὲ σημαίνει διττόν. καλοῦσι γὰρ πορίσματα καὶ ὅσα θεωρήματα συγκατασκευάζεται ταῖς ἄλλων ἀποδείξεσιν οἷον ἕρμαια καὶ κέρδη τῶν ζητούντων ὑπάρχοντα, καὶ ὅσα ζητεῖται μὲν εὑρέσεως δὲ χρῄζει καὶ οὔτε γενέσεως μόνης οὔτε θεωρίας ἁπλῆς. ὅτι μὲν γὰρ τῶν ἰσοσκελῶν αἱ πρὸς τῇ βάσει ἴσαι, θεωρῆσαι δεῖ, καὶ ὄντων δὴ τῶν πραγμάτων ἐστὶν ἡ τοιαύτη γνῶσις. τὴν δὲ γωνίαν δίχα

"Unum quid geometricorum nominum corollarium est; hoc autem duplex quidpiam significat; vocant enim corollaria quæcunque etiam theoremata, una cum aliorum demonstrationibus probantur, veluti lucra inexpectata atque emolumenta quærentium existentia: Et quæcunque quæruntur quidem, inventione autem indigent, et neque generationis solæ causa quæruntur neque simplicis contemplationis; nam quod quidem 'Æquicrurium qui ad basim sunt anguli æquales sunt,' contemplari oportet, existentiumque rerum hujuscemodi cog-

* "Age de iis etiam quæ his annexa sunt breviter differamus, quid Sumptio (λῆμμα) quid Casus, quid Corollarium (πόρισμα), quid Instantia, quid Inductio."——Baroc. Versio, p. 120.

N

" τεμεῖν ἢ τρίγωνον συϛήσασθαι, ἢ ἀφελεῖν ἢ
" θέσθαι, ταῦτα πάντα ποίησιν τινὸς ἀπαιτεῖ.
" τοῦ δοθέντος κύκλου τὸ κέντρον εὑρεῖν ἢ δυο
" δοθέντων συμμέτρων μεγεθῶν τὸ μέγιϛον καὶ
" κοινὸν μέτρον εὑρεῖν ἢ ὅσα τοιάδε μεταξύ
" πως ἐϛι προβλεμάτων καὶ θεωρημάτων. οὔτε
" γὰρ γενέσεις εἰσὶν ἐν τούτοις τῶν ζητουμένων
" ἀλλ' εὑρέσεις, οὔτε θεωρία ψιλή. δεῖ γὰρ
" ὑπ' ὄψιν ἀγαγεῖν, καὶ πρὸ ὀμμάτων ποιή-
" σασθαι τὸ ζητούμενον. τοιαῦτα ἄρα ἐϛι καὶ
" ὅσα πορίσματα Ἐυκλείδης γέγραφε, βιβλία
" προβλημάτων* συντάξας. ἀλλά περὶ μὲν τῶν
" τοιούτων πορισμάτων παρείσθω λεγειν. τὰ δὲ
" ἐν τῇ ϛοιχειώσει πορίσματα συναναφαίνεται
" μὲν ταῖς ἄλλων ἀποδείξεσιν, αὐτὰ δὲ προη-
" γουμένης τυγχάνει ζητήσεως. διον καὶ τὸ νῦν
" προκείμενον." &c.

" nitio est. 'Angulum autem bifa-
" riam secare,' vel 'triangulum con-
" stituere,' vel 'rectam lineam æqua-
" lem abscindere, vel ponere,' hæc
" omnia ut aliquid fiat postulant.
" 'Dati vero circuli centrum reperire,'
" vel 'duabus magnitudinibus com-
" mensurabilibus datis, maximam ip-
" sarum communem mensuram in-
" venire,' vel quæcunque id genus
" alia, quodammodo inter proble-
" mata atque theoremata sunt; neque
" enim quæsitorum ortus in his,
" neque sola contemplatio sed inventio
" est, Opus est siquidem quæsitum
" in conspectu et præ oculis ponere.
" Talia igitur sunt quæcunque etiam
" corollaria EUCLIDES scripsit, quippe
" qui libros corollariorum construxit.
" Verum de hujuscemodi quidem co-
" rollariis dicere prætermittatur. Quæ
" autem in elementari institutione sunt
" corollaria, simul quidem cum alio-
" rum demonstrationibus apparent,
" ipsa vero non præcipue quæruntur,
" veluti id quod in præsentia propo-
" nitur;" viz. Cor. 15. 1 Elem.

† From the sense, this word ought to be πορισμάτων, and not προβλημάτων; for whatever be the origin of that class of Propositions, there can be no doubt of their being always known and distinguished by the term πορίσματα. BAROCIUS accordingly translates it in this place by "*corollariorum*;" most probably from his having found that reading in a MS. which he relied upon. In the MS. of PROCLUS, in the Bodleian Library, which is esteemed valuable, προβλημάτων is in the text, but on the margin πορισμάτων is written in a character much resembling the text. In this sentence γ probably is omitted after βιβλία.

In this extract is a very explicit ftatement of the two very different fpecies of *Porisms*, (or *Corollaries*, as they are named by BAROCIUS,) viz. the Porifms compofed by EUCLID, a curious and difficult clafs of Propofitions, requiring inveftigation, as well as conftruction and demonftration; and the Porifms or corollaries of EUCLID's Elements, which refult from the demonftration of other Propofitions, and which often prefent themfelves unexpectedly. PROCLUS proceeds to illuftrate this latter clafs of Porifms by the example of a corollary annexed to 15 Prop. 1 Elem. the preceding extract being part of his commentary on that Propofition. He then gives a more diffufe defcription of this corollary of the elements, of which a portion is annexed.

(HERVAGII. p. 80. BAROC. p. 174.)

“ Ἔστιν οὖν τὸ πόρισμα θεώρημα διὰ τῆς ἄλλου “ προβλήματος ἢ θεωρήματος ἀποδείξεως, “ ἀπραγματεύτως ἀναφαινόμενον. ὥσπερ γὰρ “ κατὰ τύχην περιπίπτειν ἐοίκαμεν τοῖς πο- “ ρίσμασιν, οὐ γὰρ προθεμένοις, οὐδὲ ζητήσασιν “ ἀπαντᾷ, ὅθεν αὐτὰ καὶ τοῖς ἑρμαίοις “ εἰκάσαμεν, καὶ ἴσως οἱ δεινοὶ τὰ μαθημα- “ τικὰ, κατὰ ταύτην αὐτοῖς ἔθεντο τὴν ἐπωνυμίαν, “ ἐνδεικνύμενοι τοῖς πολλοῖς καὶ ἐπὶ τὸ φαι- “ νόμενον κέρδος ἐπτοημένοις* ὅτι ἄρα τὰ “ ἀληθῆ θεοῦ δῶρα καὶ τὰ ἕρμαια ταῦτα ἐστὶν “ οὐχ οἷα ἐκείνοις δοκεῖ.”

* Forfan ἡδομένοις.

“ Corollarium eft theorema quod “ ex alius problematis vel theorematis “ demonftratione ex improvifo emer- “ git: nam veluti cafu quodam in “ corollaria incidere videmur, nec “ proponentibus enim nobis, neque “ etiam quærentibus obviam fefe “ offerunt. Unde hæc quoque lucris “ affimilavimus: et fortaffe mathe- “ maticarum rerum periti hoc ipfis “ impofuere nomen, oftendentes vul- “ go, quippe quod apparenti gaudet “ lucro, quod utique vera DEI mu- “ nera, veraque lucra hæc funt, non “ autem quæ illi videntur,” &c.

Then follow feveral diftinctions of corollaries, into arithmetical and geometrical, into thofe arifing from theorems and thofe from problems, with fome others of no particular importance. Thefe extracts, it muft be allowed, are not expreffed with uniform clearnefs, but they difcriminate fufficiently the Porifms of EUCLID from the corollaries of the Elements; and they correfpond with the more general expreffions of PAPPUS on this fubject, which PROCLUS in these paffages plainly had in view. For illuftrating the

the diſtinction, he premiſes that the 5th Prop. 1 Elem. is a theorem, and requires demonſtration: alſo, that the 1ſt, 3d, and 9th Propoſitions are problems, each requiring ſomething to be done or conſtructed; he then adds, that to find the centre of a given circle, (1. 3. Elem.) or to find the greateſt common meaſure of two magnitudes, (2. 7. Elem.) or ſuch like things, are in ſome ſort of an intermediate character between problems and theorems. For the conſtruction of theſe things ſought is not given in the enunciation, but invention is requiſite for finding that conſtruction, and alſo for diſcovering the demonſtration; for it is neceſſary to exhibit to the eye the conſtruction of the things ſought. Such, ſays he, is the nature of the ſecond kind of Poriſms, of which EUCLID compoſed books.* Of theſe obſervations, not altogether clear, I have given, in a ſort of paraphraſe, the meaning as I underſtand it to be. It is plain he was acquainted with the obſervation of PAPPUS, that Poriſms are of a middle nature between problems and theorems, though ſome doubt may be entertained, whether the examples which he quotes be properly ſtated as Poriſms, as was obſerved in a former note p. 47. It may be remarked here, that PROCLUS, in referring to the 2d. 7th Elem. as a Poriſm, ſuppoſes that there may be arithmetical Poriſms as well as geometrical, and this Propoſition is altogether unconnected with *Loci*, or even with geometrical poſition of any kind.

I may obſerve further in this place, that *Loci*, which by PAPPUS are reckoned a claſs of Poriſms, have alſo ſomewhat of the intermediate character between problems and theorems; though they are generally reckoned to belong to the latter claſs. In the *Locus* the conſtruction muſt be inveſtigated, as in the *Poriſm*; and every *Locus* is eaſily convertible into a Poriſm.

The etymology of the Poriſm, or corollary of the Elements, from πόρισμα ſignifying gain, may be right; and the other meaning of the Greek term πόρισμα, implying inveſtigation, may be the ground of its being applied to the Poriſms of EUCLID. In the firſt ſenſe it is the common corollary, which is an acquiſition (or gain) from another Propoſition, from the demonſtration of which it reſults often unexpectedly.† Πόρισμα alſo, from the other ſignification of the word, properly denotes

* See Note, p. 90.

† Corollarium, aſſumed by BAROCIUS as the proper tranſlation of πόρισμα, may expreſs the firſt meaning of it, but has no connection with the other.

any thing to be investigated, which corresponds with the character of the Porisms of EUCLID.* And thus, without any connection between these two classes of Propositions, they may incidentally, from the two unconnected meanings of a Greek word, have obtained the same name.

It is necessary, however, to remark, that the first extract from PROCLUS, (p. 58, HERVAG.) containing the very short description of Porisms, does not, as it stands, accurately correspond with the detailed and more intelligible description in the second. If the word προβλημάτων in the former passage be the true reading, it might thence certainly be urged, that Porisms have a reference to problems; and an argument might be derived in favour of the opinion, that the ancient notion of Porisms was similar to that proposed by Mr. PLAYFAIR. But this preliminary description of Porisms by PROCLUS is surely too short and general to be the foundation of any such inference, without other more precise authorities; and the subsequent passage, which is much more particular and intelligible, contains no such reference to problems. Besides, it is well known, that the only printed edition of PROCLUS is very erroneous.† If it may be supposed that προβλημάτων is printed for πορισμάτων,‡ as it most certainly is in another place in the second extract, the two passages will be more consistent; and though the expression in the first be short and inexplicit, it

* Notwithstanding these notices of EUCLID's Porisms, PROCLUS does not speak of their curiosity and importance in the manner that PAPPUS does. And it is somewhat remarkable, that when PROCLUS gives a catalogue of the works of EUCLID, (p. 20, HERVAG. edit.) he makes no mention whatever of his Porisms. Dr. SIMSON, most probably, had considered these passages in his first enquiries on this subject; but as they could not give the smallest aid towards his discovery of the true nature of Porisms, he does not seem afterwards to have attended to the remarks of PROCLUS, as there is no allusion to them among his papers.

† BAROCIUS, in the dedication of his translation of PROCLUS, says, "quamvis nescius non "essem quod impressi fuerant BASILIÆ quatuor PROCLI DIADOCHI libri commentariorum in "primum Elementorum EUCLIDIS; quos adeo laceros et corruptos vidi, ut nihil boni ex iis "elicere potuerim: editi namque erant perinde ac si editi nunquam fuissent." Afterwards, having mentioned the different MSS. to which he had access, he adds, "ubi ex iis omnibus "exemplaribus quoad fieri potuit unum integrum feci quod postremo e Græca lingua in "Latinam converti, &c."——PROCLUS BAROCII, Patav. 1569.

‡ I must observe, that in the translation by BAROCIUS, which, though it includes many corrections of the printed Greek, yet contains many errors, the MS. which he relied on must have had προβλημάτων in this place, as in the edition by HERVAGIUS.

being only an introductory notice of Porifms, yet it may be underftood. I conceive, therefore, that in both extracts his object is the fame, viz. to diftinguifh the Porifms compofed by EUCLID from the Porifm which is the common corollary; and he does it on the fame principle, though more briefly in the firft than in the fecond: and if πορίσματων be affumed as the true reading in the firft paffage, it may be freely tranflated thus: "The term Porifm is applied to "that clafs of Propofitions of which are the Porifms compofed by EUCLID; "but more properly the term Porifm is applied to Propofitions not propofed by "us, but arifing from the demonftration of other Propofitions; and being thus "an unexpected acquifition- they obtained that name."

It may be further obferved, that in the Commentary of PROCLUS on the 1ft, 1. Elem. there are various diftinctions of problems ftated; one of which, viz. the πλεονάζον and the ἐλλεῖπες may here be properly mentioned. By the former term (exceeding) is meant any problem which has more conditions or data than are neceffary to the folution. As for inftance, if it be propofed to defcribe an equilateral triangle, of which each of the angles fhall be two thirds of a right angle. This laft condition is implied in the former, and is therefore fuperfluous. And if an unneceffary condition be added, the probability is that it will be inconfiftent with the others, and render the problem fo ftated impoffible. The other term (deficient) expreffes the want of a neceffary condition, by which the problem becomes, according to modern language, indeterminate.* Ἐλλεῖπες δὲ

* In PAPPUS and PROCLUS the term ἀδιόριστος, commonly tranflated *indeterminatum*, or indeterminate, is applied to problems which do not require or admit of any determination, a meaning altogether different from the modern ufe of the word: ἀόριστος is indeed ufed by PROCLUS in the modern fenfe of indeterminate. The following fentence may be quoted, HERVAGIUS, p. 61, and BAROC. 127, at the clofe of the commentary on the firft propofition of the Elements: "φανερὸν οὖν ἐκ τούτων ὅτι τὰ κυρίως λεγόμενα προβλήματα, βούλεται "τὴν ἀοριστίαν διαφεύγειν, καὶ μὴ εἶναι των, ἀπειραχῶς γιγνομένων. λεγεται δε ὅλως κακεῖνα προ- "βληματα δια την ὁμωνυμίαν τε προβληματος." &c. "Ex his itaque manifeftum eft, quod ea "quæ proprie problemata appellantur, indeterminationem effugere debent, et non effe ex "eorum numero quæ infinitis modis fiunt. Problemata tamen et illa dicuntur per problematis "æquivocationem, &c." BAROC.

In this paffage it is intimated that indeterminate Propofitions are alfo called *problems*, from a double meaning of the word; and among fuch *problems Loci* muft of courfe have a place, and therefore *Porifms* alfo; but it is not eafy to reconcile the different obfervations of PROCLUS on this fubject.

ἐστι πρόβλημα, το προσθήκης ἄλλης δεόμενον ἵνα ἐκ τῆς ἀοριστίας, εἰς τάξιν καὶ ὅρον ἐπιστημονικὸν ἀχθῆ.† HERVAG. edit. p. 61.

If any fuch notion had exifted in the times of PAPPUS or PROCLUS, as the ingenious definition of Mr. PLAYFAIR involves, we might have expected fome reference or allufion to it in a difcuffion like this concerning problems. PROCLUS, in the prefent ftate of the text, even with all the fkill of BAROCIUS, is not always intelligible, nor confiftent in his different obfervations on the fame fubject. See his comment on the 1ft Prop. EUCL.; alfo on the 22d Prop. compared with his comment on the corollary to the 15th Prop.

NOTE E. p. 57.

The 117th Prop. of the feventh book of PAPPUS has become remarkable, from the confideration which has been given to it by many diftinguifhed mathematicians of modern times, more than from any particular excellence belonging to it, as it ftands in that author. In 1742, an extenfion of it was propofed by M. CRAMER of Geneva to M. DE CASTILLON, viz. by fuppofing the three points not to be in a ftraight line, as is affumed in the Propofition of PAPPUS; and then the problem was thus expreffed, "a circle and three "points being given in pofition, to infcribe a triangle in the circle of which "the three fides fhall refpectively pafs through the three given points." It was not however till 1776, that CASTILLON publifhed a folution, in the Berlin Memoirs of that year. The folution is geometrical, and diftinguifhed into many cafes. In the fame volume is an algebraical folution by LA GRANGE, by means of fome trigonometrical formulas. In the *Petersburgh Acts* for 1780, are folutions of the fame problem by Meffrs. EULER, LEXEL, and FUSS. In the *Memorie di Mathematica e Fisica della Societa Italiana*, tom. iv. Verona, 1788, are two memoirs refpecting this problem.* The firft is by GIOR-

† HUGO DE OMERIQUE, in the fourth book of his *Analyfis Geometrica*, feems to have had fome of the diftinctions of PROCLUS in view. The *Deficiens Problema* of the latter is called *diminutum* by the former, who gives fome examples of fuch problems, in which the folution turns out to be a *Locus*.

* The following folutions and extenfions of this problem were pointed out to me by Mr. PLAYFAIR.

DANO DI OTTAJANO, in which is a ſketch of the hiſtory of the problem; and though his ſolutions are geometrical, yet he appears to have had but an imperfect notion of the ancient geometrical analyſis. He reſolves the problem before mentioned, adding a variation of it, viz. that the three lines from the angular points of the inſcribed triangle to the three given points may make angles with the ſides of the triangle equal to given angles; and concludes with an important extenſion to the caſe of a polygon, of any number of ſides, which he inſcribes in a given circle, ſo that the ſides reſpectively ſhall paſs through the ſame number of given points. The other memoir in the ſame volume is by Signor GIAN. FRANCESCO MALFATTI, and contains a good ſolution of the very general problem laſt mentioned; from reducing the caſe of polygons to that of the triangle, by the uſe of two very eaſy theorems. I ſhould have added to the account of the firſt memoir, that the author acknowledges his inability to reſolve the problem algebraically, and expreſſes a wiſh that ſome mathematician would attempt a ſolution purely analytical. Hence alſo this problem may be conſidered as an example of what Dr. SIMSON obſerves, that many geometrical problems are reſolved more eaſily by the ancient analyſis than by the modern.

In the Berlin Memoirs for 1796, is a memoir by LHUILIER, containing an algebraical ſolution of the moſt general caſe of the polygon. He premiſes two trigonometrical theorems, one of which is the baſis of La GRANGE's ſolution of the caſe of the triangle, and by means of them proceeds ſucceſſively from the ſimple to the more complex caſes of polygons; and in all the caſes deduces a quadratic equation, from the ſolution of which the problem is reſolved. The expreſſions of courſe become very complex; and a geometrical conſtruction derived from them would be not leſs ſo. LHUILIER extends the problem alſo to Conic Sections, and adds a ſimilar one reſpecting the ſphere.

I ſhall ſubjoin Dr. SIMSON's ſolution of the caſe, when the three points are not in a ſtraight line; dated Auguſt 30, 1731, many years before this problem acquired celebrity from having employed the ſkill of ſo many eminent mathematicians. The Doctor wrote all his mathematical notes in Latin, and I give this Propoſition in his own words.*

† In this and ſome other Propoſitions of Dr. SIMSON's copied into theſe notes, the Data are quoted as numbered in the old edition. I may remark alſo, that there are ſome differences in the phraſeology of Dr. SIMSON's early writings from what he uſed in more advanced life, but they are not important.

("Eſt Prop. 117, lib. vii. generalior facta.") Fig. 4.

" Datis tribus punctis A, B, C, et circulo DEF positione dato, " a duobis ex punctis A, B, inflectere ad circumferentiam AE, EB " occurrentes circulo in D, G, quæ faciant DGC, rectam lineam."

" Factum puta; et ducatur DF parallela ipſi AB junctæ, occurrens circulo " in F, et juncta FG occurrat AB in H. Quoniam igitur propter parallelas " eſt angulus BHF æqualis angulo DFG, hoc eſt, propter circulum, angulo " DEG, erunt triangula AEB, BHG æquiangula, et igitur rectangulum ABH " æquale erit dato rectangulo (92 Dat.) EBG, et datur AB, quare et BH et " punctum H, et igitur juncta HC poſitione dabitur; occurrat hæc ipſi DF " in K, et erit angulus DKH æqualis dato angulo BHK; quare eo deventum " eſt, ut a duobus punctis datis C, H, inflectantur ad circumferentiam CG GH, " ita ut DF datum faciat angulum DKH cum ipſa HC; puta factum, et " ducatur DL parallela ipſi CH, et juncta LF occurrat CH in M, ergo dabitur " punctum M, ut prius oſtenſum de puncto H; [ſc. quoniam angulus CMF " æqualis eſt angulo FLD, hoc eſt angulo FGC, igitur æquales ſunt anguli " HMF HGC, et triangula HMF, HGC, ſimilia, quare et CHM rectangulum " dato FHG eſt æquale, et datur GH, ergo et MH et punctum M,] et propter " datum angulum DKH dabitur angulus KDL, et igitur recta LF magnitudine " dabitur [88 Dat.[quoniam igitur a dato puncto M ad circulum poſitione " datum ducta eſt MFL, faciens FL quæ circulo intercepitur datam, dabitur " ML poſitione, et igitur LD et AD, et BE. Q. E. I.

" Compoſitio.

" Fiat rectangulum ABH æquale rectangulo contento ſegmentis cujuſvis " rectæ quæ a puncto B ad circulum producitur, et jungatur HC, fiat vero " rectangutum CHM æquale rectangulo contento ſegmentis cujuſvis rectæ " quæ a puncto H ad circulum producitur, et a puncto M ducatur MFL quæ " abſcindat ſegmentum LEDF capiens angulum angulo CHB æqualem, junctaque " HF occurrat circumferentiæ in G, juncta vero BG eidem occurrat in E, et " juncta AE eidem occurrat in D. Erunt puncta D, G, C, in recta linea. Jungatur " DF, et connectentur DL, DG, CG: et quoniam rectangulum ABH æquale " eſt ex conſtructione, rectangulo EBG, erit angulus BHG æqualis angulo " E hoc eſt angulo DFG, igitur parallelæ ſunt AB, DF, ergo angulus DKH

"æqualis est ipsi BHK, hoc est ex constructione angulo LDF quare parallelæ "sunt DL HK; et propter rectangulum CHM æquale rectangulo FHG in "circulo sunt puncta C,M,G,F; ergo angulus CGF æqualis est [angulo "CMF, hoc est] ipsi DLF, et propter circulum anguli DLF et DGF "simul æquales sunt duobus rectis, ergo anguli CGF, DGF, simul æquales sunt "duobus rectis, et igitur recta est linea DGC. Q. E. D."—"Transcriptum "Aug. 30, 1731."

It may also be mentioned that Dr. SIMSON at the same time gave two variations of the problem, viz. the same things being given to make the line DG parallel to a given straight line; and also the same things being supposed, to inflect AEB, so that the line DG may make an angle with the line drawn from D to the third given point, equal to a given angle.

NOTE F. p. 58.

The case of inclinations respecting the Rhombus has always been considered as an elegant geometrical Problem, and it attracted Dr. SIMSON's attention in his early study of PAPPUS. His amendment of the figure of a Lemma in COMMANDINE's PAPPUS, (Prop. 70. lib. vii. PAPPI,) which no doubt was used by APOLLONIUS in the solution of this problem, and the Doctor's corresponding solution of it, are dated Feb. 20, 1723. The following transcript was taken from a later copy without date, but is substantially the same with the original paper, with some slight amendments.

It is proper, however, to mention, that the Problem of the trisection of an arch of a circle is treated of in the iv. b. of PAPPUS; and in Prop. 31st it is proposed to be done by an *inclination* which is a solid Problem, there resolved by an hyperbola: The case of *inclinations* employed is that of a rectangular parallelogram, though the solution be the same in any other parallelogram. Of course, a Rhombus being a parallelogram, it is also comprehended in the general solution. But this case of the Rhombus is a Problem in the treatise by APOLLONIUS *De Inclinationibus*, and is plane. Dr. SIMSON, however, in his notes on this Prop. of PAPPUS shews, that when the Hyperbola is employed in the case of the Rhombus, the requisite circle is described about the vertex of the Hyperbola, and then the intersections of the Hyperbola

and circle may be determined by plane geometry, and without the description of the Hyperbola. Thus the Problem of the Rhombus, though a case of a solid Problem, as often happens, becomes plane; and is resolved in this manner by Dr. SIMSON. But the following solution of the Doctor's is better, being no doubt the APOLLONIAN, as the lemma, Prop. 70. vii. PAPPI, is employed.

"*Prop. de Rhombo cui inservit Prop.* 70. *lib. vii.* PAPPI."

["Schemate hujus sc. Prop. 70, emendato ut in Fig 6."]

"Rhombo existente, ABCD, et producta AC ad E, facere EF "datam, et quæ ad punctum B vergat. Fig. 5."

"Factum puta et producta BC ad G, angulo FCG æqualis fiat BFG, et "GE jungatur. Ergo angulus GFE, æqualis est ipsi [FCB hoc est, ipsi BCA "seu] GCE. Sunt igitur F, C, G, E, in circulo; quare angulus FGE, "æqualis est [ipsi FCE vel ei quæ ipsi deinceps est, hoc est] angulo CDB: "et in triangulis FGE, BDC, ostensus est GFE æqualis angulo DCB "æquiangula propterea sunt triangula. Ergo ut BC ad BD, ita EF ad "EG, et data ex hypothesi est FE, quare data est EG, sive ipsi æqualis "FG. Est autem propter æquiangula triangula BGF, FGC, BG ad GF "ut GF ad GC; igitur rectangulum BGC æquale est quadrato ex GF: "datum proinde est BGC rectangulum, et data est BG ergo [84 Dat.] "dabitur CG, et punctum G propterea datum erit; et data est GF magni- "tudine, et CD positione ergo (31 Dat.) positione data est GF et punctum "F, sed et punctum B, ergo recta BFE positione data est.

"Componetur ita: Sit BH data recta, et ut BC ad BD, ita fiat BH ad "quartam BK: ad datua vero rectam BC applicetur rectangulum BGC "æquale quadrato ex BK, excedens quadrato; factaque GL ipsi BK "æquali, centro G intervallo GL describatur circulus qui rectis AC, CD "occurrat in E, F, punctis: Erit juncta EF æqualis datæ BH vergetque ad "punctum B.

"Jungantur enim GE et GF; et quoniam GL media est proportionalis "inter BG, GC, erunt puncta E, F, B, in recta linea per Prop. 70. lib. vii. "PAPPI. Et quoniam est BG ad GL seu GF ut GF ad GC, æquiangula erunt "triangula BGF, FGC; quare angulus BFG æqualis est ipsi FCG, et angulus

"GFE ipſi [FCB hoc eſt, ipſi BCA ſeu] GCE. Ergo in circulo ſunt puncta F, "C, G, E, angulusque FGE æqualis erit [ipſi FCE vel ei qui ipſi deinceps eſt "hoc eſt] angulo CDB: æquiangula igitur ſunt CBD, EFG triangula, et ut "CB ad BD, hoc eſt ut BH ad BK, ita erit EF ad EG vel FG; æqualis autem "eſt BK ipſi FG ſeu GL, ergo æqualis eſt BH ipſi EF. Q. E D.

"Potuiſſet compoſitio æque ac analyſis facta fuiſſe ſine PAPPI Lem-"mate, ſc. jungendo BF et producendo eam ad AC, in E; ita enim non opus "fuiſſet Lemmate ad oſtendendum puncta E, F, B, in recta linea eſſe. Hæc "autem compoſitio ſtricte loquendo reſpondiſſet problemati quo requiritur a "puncto B ducere rectam BFE, et facere EF datam; et non huic quo re-"quiritur producere AC ad E, et facere EF datam quæ ad punctum B vergit; "hoc eſt in angulo exteriore Rhombi ECF, ſeu interiore ACF, aptare rectam "EF magnitudine datam quæ vergat ad punctum B; et ut compoſitio huic "enunciationi ad amuſſim reſpondeat videtur Lemma PAPPI utile, et non "aliam ob cauſam. R. S."

"In caſu anguli interioris Rhombi, addatur determinatio ope, Prop. 73, 74, "lib. vii. PAPPI, et fiant figuræ reſpondentes pro caſu, [Prop. 70. PAPPI, "et inde in problemate] quando ponenda EF in angulo interiore ACD, et "etiam per B punctum; [vel ad B vergens.]

"Lemma in linea ultima, fol. 205, a. PAPPI, (COMMAND. 1589) aſſump-"tum ab eo, eſt ut ſequitur. Vid. fig. Prop. 70. 'Si a puncto C in diametro 'DF ducantur ad circumferentiam rectæ CL CK, quæ faciant cum diametro 'æquales angulos LCF, KCF, erunt ductæ inter ſe æquales;' cujus demon-"ſtratio facilis eſt. Vide fig. 6. [Schema emendatum, Prop. 70. lib. vii. PAPPI."]

NOTE G. p. 62.

As Dr. SIMSON had at one time intended to compoſe a treatiſe on *Loci ad ſuperficiem*, it may be ſatisfactory to ſome readers to ſee his preface, in which he expreſſes this intention; and this, with ſome excerpts from the remaining fragments on this ſubject, may give a general notion of the Doctor's views of this ſubject.

"De *Locis ad ſuperficiem*, quæ ab EUCLIDE aliiſve geometris antiquis "tradita fuere, injuria temporis deperdita ſunt, exceptis paucis iis quæ apud

"PAPPUM habentur. Recentiores* autem nihil de *Locis* hisce cognoscere "videntur, exceptis *Locis* quæ sunt ad superficiem rotatione alicujus ex sec-"tionibus conicis circa diametros vel alias rectas genitam, cum tamen "magnus sit numerus aliorum qui hoc modo minime formari possunt. Operæ "igitur pretium facturum ab eruditis videri spero, si viam qua *Loci* hi investi-"gari possunt exemplis quibusdam patefaciam, quibus rite intellectis, non "difficile erit ei qui doctrina *Locorum solidorum* probe instructus fuerit, "*Locos* ad quamvis superficiem, quæ quovis modo ex sectionibus conicis "vario situ dispositis oriri potest, investigare et describere.

"Hi autem omnes [phrasi algebraica] sequente æquatione continentur, viz.

$$\text{``}\, xx+\frac{a}{b}xy+ax+by+\frac{a}{c}yy+\frac{a}{d}yz+\frac{a}{e}xz+gz+\frac{b}{k}zz = \text{Dato.}$$

"Signis utcunque mutatis."

The first example, as was observed in the Memoir, is unfinished. The Doctor had proceeded so far as to ascertain the common sections of the superficies required with several planes; some others were proposed, but not investigated, and there is no trace of his having resumed the inquiry. The portion of this example which is written becomes not a little complex, requiring also subsidiary Propositions; and therefore the enunciation only of the *Locus* is here stated; which may be referred to the diagram of the subsequent Proposition. [Fig. 8.]

"Data sint in plano quovis, puta in plano chartæ in quo "schema est, duo puncta A, B, et juncta BA, in ea sumatur "quodvis punctum D, et ipsi AD ad rectos angulos, ducatur in "eodem plano recta DE, et a termino ejus E ad planum erigatur "ad rectos angulos EF, sitque rectangulum BDA simul cum "rectangulo DEF, et quadrato quod fit ex EF æquale spatio dato, "puta (simplicitatis gratia) quadrato ex ipsa AB: quæritur "superficies in qua omnia puncta F versantur."

To this no equation is annexed.

* "N. B. scripta fuerunt hæc priusquam librum, D. N. *des Courbes a double Courbure* "vidissem, itaque mutanda est hæc introductio."

The following is a short example of a *Locus ad superficiem*, found among several unfinished sketches of such Propositions which were mentioned in the Memoir. It is given as it stands in the Doctor's own words.

"*Locus ad Superficiem.* $xx + \frac{ay^2}{b} + \frac{az^2}{c} = d^2$ [FIG. 8.]

"Data positione recta linea AB, punctoque in ipsa A, in "dato positione plano ABC; si a puncto D in recta AB ipsi "ad rectos angulos ducatur recta DE, et a termino ipsius E, "erigatur ad planum ABC perpendicularis recta EF; fueritque "quadratum ex AD, simul cum spatio quod ad quadratum "ex DE, et eo quod ad quadratum ex EF, datas habent rationes, "æquale dato spatio, scil. quadrato ex data recta AB. Tanget "punctum F, superficiem positione datam."

"Sit enim quadratum ex DG, id quod ad quadratum ex DE datam "habet rationem, et juncta AG occurrat circulo centro A semidiametro AB "descripto in H, idem vero circulus occurrat DE in K, L, punctis. Quoniam "igitur quadrata ex AD, DG, hoc est quadratum ex AG, una cum eo quod "ad quadratum ex EF datam habet rationem æquale est quadrato ex AB, seu "AH, auferatur commune quadratum ex AG, et reliquum rectangulum KGL "æquali erit spatio quod ad quadratum ex EF datam habet rationem. Fiat "autem ut DG ad DE, ita DK ad DM; quadrata igitur ex hisce rectis pro-"portionalia erunt, et [per. 19. 5.] quoniam quadratum ex DK est ad quadratum "ex DM ut quadratum ex DG ad quadratum ex DE, erit in eadem ratione "data reliquum rectangulum KGL ad excessum quadratorum ex DM, DE, "hoc est, facta DN æquali ipsi DM, ad rectangulum MEN. Datur igitur "ratio rectanguli MEN ad rectangulum KGL, hujus autem ratio ad qua-"dratum ex EF datur, quare [per 8. Dat.] datur ratio MEN ad quadratum ex "EF. Quoniam vero datur ratio DK ad DM, tangit autem punctum K "circumferentiam positione datam, semidiametro, scil. AB descriptam, tanget "punctum M ellipsin positione datam, cujus semiaxis major est ipsa AB; "occurrat vero EF sphæroidi oblongæ hac ellipsi descriptæ in O. Et quoniam "data ostensa fuit ratio rectanguli MEN, hoc est quadrati ex EO ad quadratum "ex EF, dabitur ratio EO ad EF. Si igitur altitudo EO omnium punctorum "O hujus sphæroidis supra planum ABC diminuatur, vel augeatur, in ratione

" data EO ad EF ſuperficies ſolidi, diminutione hac vel additione producti erit " Locus punctorum F. Q. E. I."

Another ſhort Propoſition exhibits a method of deſcribing the Caſſinian curve by means of *Loci ad ſuperficiem.*

PROB. [Fig. 9.]

" Describere curvam Cassinianam."

" Sint A, B, Foci, C centrum, et D junctum in curva; igitur junctis AD " BD, erit rectangulum ADB æquale dato ſpatio."

" Plano in quo eſt curva ducatur ad rectos angulos recta DE ipſi DA " æqualis, et tanget punctum E conum rectangulum cujus vertex eſt punctum " A, axis vero plano ABD ad rectos angulos; et quoniam rectus eſt angulus " EDB, et rectangulum EDB æquale dato ſpatio, tanget idem punctum E " hyperbolam æquilateram cujus una aſſymptotos eſt DB, altera vero recta " BF quæ ad planum ADB eſt perpendicularis. Revolvatur igitur hyperbola " hæc circa BF tanquam axem, et deſcribetur ſolidum hyperbolicum acutum; " communis vero ſectio ſuperficiei hujus cum ſuperficie coni prædicti, erit linea a " cujus punctis ſi ducantur perpendiculares ad planum ADB, ipſi occurrent " in punctis quæ ſunt in curva Caſſiniana, ut patet."

Another example may be the 28 Prop. lib. iv. PAPPI, as corrected by Dr. SIMSON.

PROB. V. PROP. 28. Lib. iv. PAPPI. [Fig. 7.]

" Hic igitur lineæ ortus magis mechanicus est, ut dictum " fuit, geometrice vero per Locos qui ad superficies " dicuntur"* " resolvi potest hoc modo."

" Sit circuli quadrans poſitione datus ABC, et ducatur, ut contingit, recta " linea BD, et ad BC perpendicularis EF quæ ad circumferentiam DC pro- " portionem datam habeat. Dico punctum E ad lineam eſſe.

* Deeſt hoc verbum in MS. BULL.

"Intelligatur enim a circumferentia ADC recti cylindri ſuperficies, et
"in ipſa linea ſpiralis [CGH] deſcripta, [incipiens a puncto C, ita ut
"velocitas puncti deſcribentis in recta CM quæ perpendicularis eſt ad
"planum ACB, ſit ad velocitatem ipſius CM in quadrante circuli CDA in
"data ratione quam habet EF ad circumferentiam DC, quæ propterea
"ſpiralis] poſitione data erit. Sitque HD latus cylindri, et ad planum
"circuli perpendiculares ducantur EI, BL, et per H ipſi BD parallela
"ducatur HL, [ipſi EI occurrens in I.] Itaque quoniam proportio rectæ lineæ
"EF ad circumferentiam DC eadem eſt quæ proportio EI, hoc eſt DH
"ad eandem DC propter ſpiralem; erit recta EF ipſi EI æqualis. Suntque
"FE, EI, poſitione, ergo et juncta FI poſitione erit, [N. B. Non dicit FE, EI,
"poſitione datas, datum non eſt punctum E, ſed intelligit tantum angulum
"EFB datum eſſe, et EI perpendicularem eſſe ad planum circuli, unde
"quoniam rectus eſt angulus EFB, erit:] et FI ad BC perpendicularis. Eſt
"igitur FI in plano [ſc. quod per BC tranſit, et per dimidium anguli recti
"inclinatum eſt ad planum ſubjectum ABC, angulus enim EFI dimidium
"eſt recti] quare et punctum I eſt in eodem; atque idem punctum eſt in
"ſuperficie, fertur enim HL et per lineam ſpiralem CGH, et per rectam
"lineam BL, et ipſam poſitione datam, ſemper exiſtens [ſc. ipſa HL] parallela
"ſubjecto plano; ad lineam igitur eſt punctum I, [ſc. ad communem ſectionem
"ſuperficiei a recta HL deſcriptæ, cum plano prædicto quod per BC tranſit]
"ergo eſt punctum E ad lineam [quæ ſc. formatur ducendo perpendiculares
"a linea in qua verſatur punctum I, ad ſubjectum planum.] Hoc quidem
"univerſe reſolutum eſt, [ſc. quæcunque fuerit data ratio EF ad DC.] Si
"autem hæc ratio æqualis fuerit rationi quam habet BA ad circumferentiam
"quadrantis ADC, prædicta linea [in qua eſt punctum E] quadratrix efficitur."

The 29th Prop. next following is, "Poteſt etiam illud per lineam ſpiralem in plano deſcriptam reſolvi ſimili ratione."—It is unneceſſary in this place to give the inveſtigation, but I ſhall remark ſome of the moſt material corrections of it by Dr. SIMSON.

In line 6. fol. 59. a. COMMAND. "in ſuperficie cylindrica, in qua eſt linea "ſpiralis."—Dr. SIMSON ſuppoſes from Note F on this Propoſition, that in the MS. uſed by COMMANDINE the reading was ἐν κυλινδροείδει ἄρα ἐπιφανεία, which is more correct, and is alſo confirmed by the MS. BULL. It ſhould therefore have been tranſlated "in ſuperficie cylindroeide."—Alſo, inſtead

of "in qua eſt linea ſpirali," it ſhould be "quæ formata eſt a linea ſpirali:" for in MS. BULL it is "τῇ ἀπὸ τῆς ἕλικος," which is plainly the true reading.

Lin. 8. "ad lineam eſt ipſum K," add, "ſc. ad eam lineam quæ eſt "communis ſectio ſuperficiei cylindroeides et ſuperficiei conicæ." Lin. 11. for "ergo K et I ſunt in ſuperficie," read, "ergo eſt punctum I in ſuperficie, ſc. "in ea quam vocavit πληκτοειδοῦς." Lin. 13. for, "ad rectam igitur lineam eſt "punctum I;" read, "ad lineam igitur eſt punctum I,* "ſc. eam quæ com- "munis eſt ſectio ſuperficiei πληκτοειδοῦς† cum plano quod eſt ſuper rectam BC, "et ad ſubjectum planum eſt inclinatum per dimidium recti."

NOTE H. p. 65.

For the ſatisfaction of ſome readers, I ſhall inſert Dr. SIMSON's Paper on Series, encloſed in his letter to Dr. JURIN, of Feb. 1, 1723, (for which ſee note p. 65.) The two Preliminary propoſitions are very eaſy, and are now to be found in all the compleat treatiſes of trigonometry; but as they are ſhort, they are inſerted as they ſtood in Dr. SIMSON's original paper.

Dr. SIMSON about a month after received Dr. JURIN's anſwer, (of March 5, 1723,) communicating Mr. MACHIN's ſerieſes; and to his paper he has added a note, ſtating the correſpondence of five of his own ſerieſes with thoſe of Mr. MACHIN's. Mr. MACHIN communicated ſeven; and two of them, which had not before occurred to Dr. SIMSON, were readily inveſtigated by him.

"PROP I. Fig. 11. Sint in circulo cujus centrum eſt C duo quilibet arcus AB, "AD, qui ſimul ſumpti ſunt quadrante BL minores; erit exceſſus quadrati quod "fit a ſemidiametro ſupra rectangulum ſub tangentibus arcuum AB, AD ad "quadratum a ſemidiametro, ut tangentes arcuum AB, AD ſimul ſumpti, ad "tangentem ſummæ arcuum.

"Sint enim AE, AF tangentes arcuum AB, AD, et ſit BG, tangens ſummæ "ipſarum BD, et juncta CL, occurrat ipſi AF in H, ducatur vero FK parallela "ipſi AC, et occurrens EC in K, et jungatur KH. Igitur quoniam anguli "HCK, HFK recti ſunt, erunt puncta H, F, C, K, in circulo; ergo angulus "FHK equalis eſt ipſi FCE, et proinde ſimilia ſunt triangula rectangula

† For ſome obſervations on πληκτοειδοῦς, ſee Appen. II.

* This is confirmed by MS. BULL, which has πρὸς γραμμῆς ἄρα τὸ ι.

" HFK, CBG. Eſt autem ratio EF ad BG compoſita ex rationibus EF ad FK " et FK ad BG; *i. e.* ex rationibus EA ad AC, et FH ad CB ſeu AC; ergo EF " eſt ad BG ut rectangulum EA in FH, ad quadratum ex AC; ſed eſt EA in FH " exceſſus rectanguli EAH, *i. e.* quadrati ex AC, ſupra rectangulum contentum " tangentibus EA, AF, et eſt hic exceſſus ad quadratum ex AC ut EF ad BG. " *Q. E. D.*

" Cor. 1. Hinc ſi arcus AB, AD, æquales fuerint, erit exceſſus quadrati a " ſemidiametro ſupra quadratum tangentis arcus cujusvis octante circumferentiæ " minoris ad quadratum a ſemidiametro, ut tangens duplicatus ad tangentem " arcus duplicati.

" Cor. 2. Hinc etiam inveniri poteſt tangens arcus qui multiplex eſt " arcus alterius, cujus tangens datur. Si *a* tangens arcus A, et erit tangens " arcus $n \times A$.

$$\frac{na - \frac{n.\overline{n-1}.\overline{n-2}}{1.\ 2.\ 3.}a^3 + \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}}{1.\ 2.\ 3.\ 4.\ 5.}a^5 \text{ \&c.}}{1 - \frac{n.\overline{n-1}}{1.\ 2.}a^2 + \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}}{1.\ 2.\ 3.\ 4.}a^4 \text{ \&c.}}$$

" Numeratoris hujus ſeriei ſi *n* ſit numerus par, ſumendi ſunt tot termini " quot ſunt unitates in $\frac{1}{2}n$; denominatoris vero tot quot ſunt unitates in $\frac{1}{2}n+1$. " Si vero fuerit *n* numerus impar, tam numeratoris quam denominatoris " ſumendi ſunt termini $\frac{n+1}{2}$. Hanc feriem video apud Christ. Wolfium, " ex ſinubus derivatum.

" Prop. 2. fig. 12. Sint duo quilibet arcus AB, AD, quorum major non " excedit quadrantem; erit quadratum a ſemidiametro una cum rectangulo " ſub tangentibus arcuum ad quadratum ex ſemidiametro, ut differentia tangen- " tium ad tangentem differentiæ arcuum.

" Simili prorſus modo quo precedens demonſtratur.

" Cor. 1. Hinc ſi arcus minor AB, æqualis fuerit dimidio quadrantis, erit " radius una cum tangente majoris arcus ad ipſorum differentiam, ut radius ad " tangentem arcus quo AD ſuperat dimidium quadrantis. Si vero arcus major " AD æqualis fuerit dimidio quadrantis; erit radius una cum tangente minoris " arcus, ad ipſorum differentiam, ut radius ad tangentem arcus quo dimidium " quadrantis ſuperat arcum AB, *i. e.* in utroque caſu erit ſumma tangentium ad " ipſorum differentiam ut radius ad tangentem differentiæ arcuum. Corolla- " rium vero hoc, ni fallor, jamdudum innotuit.

"Præcedentium ope innumeræ ſeries exhiberi poſſunt ad longitudinem cir-
" cumferentiæ inveniendam. Plurimæ vero, ad uſum paratiores, in ſequente
" ſerie continentur. Sc. ſint tangentes $\frac{1}{a}$, $\frac{1}{2^n a}$ et ſit A arcus cujus tangens eſt $\frac{1}{a}$
" et erit, exiſtente radio $=1$,

$$\left.\begin{array}{l} \frac{1}{a} - 1 \times \overline{\beta + 2\gamma + 4\delta + 8\varepsilon}\ \text{\&c.} \\ -\frac{1}{3} \times \frac{2^n}{2^n a^3} + \frac{1}{3} \times \overline{\beta^3 + 2\gamma^3 + 4\delta^3 + 8\varepsilon^3}\ \text{\&c.} \\ +\frac{1}{5} \times \frac{2^n}{2^n a^5} - \frac{1}{5} \times \overline{\beta^5 \times 2\gamma^5 \times 4\delta^5 \times 8\varepsilon^5}\ \text{\&c.} \\ -\frac{1}{7}\ \text{\&c.} \end{array}\right\} = \text{Arcui A.}$$

$$\text{Eſt vero}\ \left.\begin{array}{l} \beta = \frac{1}{\overline{4aa \times 3} \times a} \\ \gamma = \frac{1}{\overline{16aa+3} \times 2a} \\ \delta = \frac{1}{\overline{64aa+3} \times 4a} \\ \varepsilon = \frac{1}{\overline{256aa+3} \times 8a} \\ \text{\&c.} \end{array}\right\}$$

Terminorum autem β, γ, δ, &c. tot ſumendi ſunt, quot ſunt unitates in numero integro n.

" Facile deducitur hæc ſeries ex Cor. 1 et 2 Prop. 1. et ex Prop. 2. Sit jam
" arcus D datum habens tangentem, datamque rationem ad circumferentiam;
" Sc. ſit $2\,r\,\text{D} = \text{circumf.} = c$; ſitque mA is arcus qui vel proxime excedit arcum
" D, vel ab ipſo proxime deficit exiſtente m numero integro; et inveniatur
" tangens arcus mA ope Cor. 2. Prop. 1. deinde inveniatur tangens differentiæ
" arcuum mA & D ope Cor. 1. Prop. 2. Sit tangens hic t, et erit arcus cujus
" eſt tangens, $t - \frac{1}{3}t^3 + \frac{1}{5}t^5$ &c. ut notum eſt; vocetur hic arcus B, et erit
" $m\text{A} \mp \text{B} = \text{D}$, unde $rm\text{A} \mp r\text{B} = r\text{D} =$ circumferentiæ circuli cujus diameter
" $=1$.

Ex. 1. Sit $a=4$, $n=0$, $r=4$, $m=3$, eritque $t=\frac{5}{99}$.

$$\text{Et}\quad rmA+rB=\frac{12}{4}+\frac{20}{99}$$
$$-\frac{1}{3}\times\overline{\frac{12}{4^3}+\frac{20^3}{99^3}}$$
$$+\frac{1}{5}\times\overline{\frac{12}{4^5}+\frac{20^5}{99^5}}$$
&c.*

N. B. Quoniam $\frac{1}{4}$ est tangens arcus A erit tangens arcus m A. *i. e.* 3 A $=\frac{47}{52}$ unde tangens arcus $45^\circ-3$ A erit $\frac{5}{99}$.

Ex. 2. Sit $a=5$, $n=1$, $r=4$, $m=4$; eritque $\beta=\frac{1}{515}$; $t=239$.

$$\text{Et}\quad rmA-rB=\frac{16}{5}-\frac{16}{515}-\frac{4}{239}$$
$$-\frac{1}{3}\times\overline{\frac{32}{10^3}-\frac{16}{515^3}-\frac{4}{239^3}}$$
$$+\frac{1}{5}\times\overline{\frac{32}{10^5}-\frac{16}{515^5}-\frac{4}{239^5}}$$
&c. &c. &c.

Ex. 3. Sit $n=o$ manentibus cæteris.

$$\text{Tunc}\quad rmA-rB=\frac{16}{5}-\frac{4}{239}$$
$$-\frac{1}{3}\times\overline{\frac{16}{5^3}-\frac{4}{239^3}}$$
$$+\frac{1}{5}\times\overline{\frac{16}{5^5}-\frac{4}{239^5}}\ \dagger$$

"Et manifestum est series has celerius convergere quo major est n, terminos "vero ipsarum, propter eandem causam, complexiores evadere, et proinde plus "laboris ad ipsos computandos requiri, ut ex duabus ultimo præcedentibus "seriebus liquet.

* "This is Mr. MACHIN's 4th, in the paper he sent me down in Dr. JURIN's Letter of "the 5th of March 1723."

† "Quæ est series optima a doctissimo Dom. MACHIN inventa." Et est ejus 6ta.

Ex. 4. Sit $a=2$, $n=o$, $r=4$, $m=2$. Eritque $t=\beta=\frac{1}{7}$

$$\text{Et } rmA - rB = \frac{8}{2} - \frac{4}{7}$$
$$- \frac{1}{3} \times \overline{\frac{8}{2^3} - \frac{4}{7^3}}$$
$$\times \overline{\frac{1}{5}} \times \text{\&c.}$$

This is Mr. MACHIN's 2d.

Ex 5. Sit $a=3$. Cæteris manentibus.

$$\text{Eritque } rmA + rB = \frac{8}{3} + \frac{4}{7}$$
$$- \frac{1}{3} \times \overline{\frac{8}{2^3} + \frac{4}{7^3}}$$
$$+ \overline{\frac{1}{5}} \times \text{\&c.}$$

This is Mr. MACHIN's 3d.

Ex. 6. Denique, fit $a=2$, $n=o$, $r=4$, $m=1$, eritque $t=\frac{1}{3}$.

$$\text{Et } rA + rB = \frac{4}{2} + \frac{4}{3} - \frac{1}{3} \times \overline{\frac{4}{2^3} + \frac{4}{3^3}} + \frac{1}{5} \times \overline{\frac{4}{2^5} + \frac{4}{3^5}} \text{ \&c.}$$

Quæ omnium eſt ſimpliciſſima. Eſt 1ma Dom. MACHIN.

Similiter poſito $r=6$, derivantur aliæ ſeries, ut ſi fit

$$a=\sqrt{3},\ n=1,\ m=2. \quad \text{erit } t=\frac{1}{15\sqrt{3}}.$$

$$\text{Et } rmA - rB = 2\sqrt{3} - \frac{2\sqrt{3}}{5\times 3}$$
$$- \frac{1}{3} \times \overline{\frac{\sqrt{3}}{2\times 3} - \frac{2\sqrt{3}}{5^3\times 3^4}}$$
$$+ \frac{1}{5} \times \overline{\frac{\sqrt{3}}{2^3\times 3^2} - \frac{2\sqrt{3}}{5^5\times 3^7}}$$
$$\text{\&c.}$$

"Convergit hoc duplo celerius ſerie ſimplici ex tangente $\frac{1}{\sqrt{3}}$ derivata, ſed plus "quam duplo labore."*

Theſe few notes were added by Dr. SIMSON to his original paper, after he received Mr. MACHIN's ſerieſes.

Above thirty years after this correſpondence, Dr. SIMSON gives ſome account of it to the late Earl STANHOPE, in a letter dated Jan. 9, 1758, of which the following extract will not be unacceptable.

After acknowledging ſome ingenious communications of ſeries from Lord STANHOPE, he adds: "In the beginning of the year 1723 I ſent up to Dr. "JURIN, Secretary to the Royal Society, a method of deriving ſeries for finding "the circumference of the circle from the ſeries $1 - \frac{1}{3} + \frac{1}{5}$ &c. which by itſelf "is of no uſe. I had before this deſired him to inform me if he knew if any "application had been of this ſeries for finding of others that converged quickly; "he wrote to me that he knew of none, and deſired me to ſend up the paper, "which I did, and gave ſeven ſeries as examples of the method. He wrote me "in anſwer, that on communicating it, he found that Mr. MACHIN had in the "year 1705 or 1706, given in a paper to the Royal Society upon this affair, "which he afterwards took back again; but that he had prevailed on Mr.

* "In the paper wrote and ſigned by Mr. MACHIN, which Dr. JURIN ſent me incloſed "in his letter of the 5th of March 1723, there are ſeven ſerieſes ſet down for finding the "arch of 45°, the

"1ſt of which is the ſame with the 6th in this letter; and the
"2d is the ſame with the 4th of this; and the
"3d is the ſame with the 5th of this; and the
"4th is the ſame with the 1ſt of this; and the
"6th the ſame that is here the 3d.

"Mr. MACHIN's 5th is for the arch of 45°. as all his others are, when radius is 1.

$$\frac{3}{4} + \frac{1}{20} + \frac{1}{1985} - \frac{1}{3} \times \overline{\frac{3}{4^3} + \frac{1}{20^3} + \frac{1}{1985^3}} + \frac{1}{5} \times \text{\&c.}$$

"His 7th is $\frac{8}{10} - \frac{1}{100} - \frac{11}{5637} + \frac{10893}{41480222056636}$

$$- \frac{1}{3} \times \overline{\frac{8}{10^3} - \frac{1}{100^3} - \frac{11^3}{5637^3} + \frac{10893^3}{41480222056636^3}} + \text{\&c.}$$

"Vide the inveſtigation of the 5th and 7th ſeries (of Mr. MACHIN) in a MS. book."

"MACHIN to ſend me a ſhort account of it, which he did in a paper ſigned "by him, which Dr. JURIN incloſed in his letter to me. It contained ſeven "ſeries, two of which were different from thoſe I had ſent, the reſt were the "ſame. In caſe your Lordſhip has not ſeen theſe ſeries, I ſhall ſet down two "of them."

Dr. SIMSON then ſtates the ſeries of Mr. MACHIN, No. 5, in the preceding note; and alſo one which he had ſent to Dr. JURIN, and was not returned by Mr. MACHIN, viz. Ex. 2d. in the preceding paper. Of this laſt he obſerves, "This ſeries is one of mine, and converges very quickly; and becauſe one part "of it conſiſts of powers of $\frac{1}{10}$, is raiſed with little more trouble than Mr. "MACHIN's, by the powers of $\frac{1}{5}$ and $\frac{1}{239}$, which was publiſhed in Mr. "JONES's *Synopsis*. It gives the circumference to the diameter 1." He adds, "The ſeries which Mr. EULER gives in page 107, vol. i. of his *Introductio in "Analysin Infinitorum*, (Lauſan. 1748,) is one of thoſe which I ſent up, and "which Mr. MACHIN ſent down, [being Ex. 6. before mentioned] but it does "not converge ſo quickly as the foregoing two." &c.

In the remainder of this long letter are ſome other obſervations on this and ſome other matters in the work of EULER referred to. But the inſertion of them is unneceſſary, ſince of theſe methods of ſquaring the circle by Mr. EULER, there is a full and explanatory detail, with many judicious remarks and improvements by Mr. BARON MASERES, in his great repoſitory of curious mathematical tracts, the *Scriptores Logarithmici*, vol. iii. p. 169, &c.†

† It may be proper to refer the reader to ſeveral notices of theſe ſerieſes of Mr. MACHIN's, in Baron MASERES's *Eſſay on the Negative Sign*; alſo in his *Scriptores Logarithmici*, vol. iii. p. 157; and in Dr. HUTTON's *Menſuration*. Dr. HUTTON's explanation of Mr. MACHIN's ſeries, publiſhed by Mr. JONES, is reprinted in the ſame volume of *Script. Log.* with another tract alſo of the Doctor's on the ſame ſubject, p. 207, which had been publiſhed in the *Philoſophical Transactions*, and contains ſome curious diſcuſſions reſpecting ſuch ſerieſes, and ſome important improvements.

Note I. p. 69.

In a letter to Earl STANHOPE, of 22d of March 1751, he mentions very particularly the decline of his memory, and his inability to attempt ſome things recommended to him by his Lordſhip. "Perſons of my age (now paſt ſixty-"three) generally loſe the ability they had when younger, of a quick and ready "imagination; and their memory, (which, in my opinion, is either the imagi-"nation of ſenſations paſt, or the recalling imaginations we had formerly) "manifeſtly decays; and ſo far with me, that I have oftentimes difficulty to "recall thoſe I had the laſt hour, or even a few minutes before. And in long "inveſtigations, where it is neceſſary to look back a good way, this inability is "moſt eaſily obſerved, eſpecially when moſt of the ſteps are not wrote down; for I "remember ſince I could go through a longer ſeries of ſteps without writing, "than I can now well do with the help of it.* This, my Lord, makes me "afraid that I ſhall not be able to engage in the undertaking you are pleaſed to "recommend to me, and which, indeed, would be very agreeable to me; the "applying the method of the ancients to the modern inventions, ſo as they "might be demonſtrated in ſuch a way as would (to uſe your Lordſhip's juſt "and elegant deſcription of accuracy and ſtrictneſs) convince an EUCLID, an "ARCHIMEDES, or an APOLLONIUS, riſen from the grave, to examine "them. My ſcholars, Mr. MOOR, Mr. WILLIAMSON, and particularly "Mr. STEWART of Edinburgh, I hope may be able to do ſomething this "way; and I ſhall not fail to recommend it to them, and direct them as far "as I can."——"I hope, if GOD grant me health, I may do ſomething "towards reſtoring ſome pieces of the ancients; the firſt ſix, and the eleventh "and twelfth books of EUCLID's *Elements* are near ready, excepting the "figures. Next to this, I have in moſt forwardneſs APOLLONIUS *de Sectione* "*Determinata*; but PAPPUS, though I have done a good deal towards reſtoring "him, is ſo large, that it frightens me to meddle with him."——Towards the end of this letter he adds, "And I hope to do EUCLID juſtice, if it pleaſe "GOD to give me opportunity to publiſh the eight books of his *Elements*, "which want of ſecurity from being pirated does yet delay."

* I ſhall mention in this place an obſervation of Dr. SIMSON on mathematical enquiry, which, though not immediately connected with the ſubject of this letter, may gratify the

NOTE K. p. 72.

Among the fmall remains of Dr. SIMSON's mathematical correfpondence which have been preferved, are two letters from the late GEORGE LEWIS SCOTT, efq; Commiffioner of Excife, with copies of two from Dr. SIMSON in reply. The general object of thefe letters was mentioned in the memoir, and for the fatisfaction of mathematical readers, the portions of them refpecting the comparative merits of the ancient and modern analyfis are fubjoined. Mr. SCOTT's firft letter is dated London, 12th April 1764, and after fome obfervations on Dr. SIMSON's edition of the *Data*, which the Doctor had folicited from him, and in which he difcovers fome prejudice againft that book, thinking its ufe to be rather logical than mathematical, he adds:

" To be more particular, I obferved that the 27 firft Propofitions relating " to magnitudes and their ratios merely, might be much more concifely treated, " and receive an additional evidence from the algebraic method. I fhall give " but one inftance from Prop. xvii. which is the xith in BARROW, and is " by him left in darknefs; you have given it light; but would it not be fhorter " and clearer to ftate it and its demonftration in fymbols?

" The firft hypothefis, that $x-a:y::m:n$ where a, m, and n, are data. " The thefis that $x-\text{dat}:x+y$, in ratione data. The hypothefis may be " changed into

" $x-a:y::a:b$ therefore $x-a+y:x-a::a+b:a::a:\frac{a^2}{a+b}$.

" Therefore by 12.5th Elem. $x-a+y:x-a::x+y:x-a+\frac{aa}{a+b}$

" and $x-a+y:x-a::a+b:a$

" Therefore $x-a+\frac{a^2}{a+b}:x+y::a:a+b$. *q.d.e.*

" The fecond hypothefis is that $x-a:x+y::a:b$. which may be fhewn in " like manner.

reader. It is alfo in a letter to Earl STANHOPE, April 9, 1753. "What your Lordfhip " fays of the ufefulnefs, and often the neceffity, of ufing the method of induction, I have " frequently had the experience of; and though it be a proof of the weaknefs of the human " mind, it is at the fame time a good help to the finding out whether a Propofition be true or " not, and moft powerfully excites us to fearch after a ftrict demonftration."

a

"I have followed your reasoning; mine is essentially the same; but is not "the distinction by symbols of the indeterminate from the determinate or "given quantities or magnitudes concerned in the question of use? and is not "the conciseness of the expression of great use also? Many of the Propositions "of the data might be thereby reduced to intuitive evidence. In this particular "demonstration, it is shewn at once, what magnitude is to be taken from x "(viz. $a - \frac{a^2}{a+b}$), that the remainder may be in a given ratio to $x+y$. I men-"tion these particulars only to introduce a more general observation, which is, "that wherever magnitudes and their ratios are considered, the method of the "ancients seems inferior to that of the moderns, with respect to facility, though "not at all so in point of accuracy. When I look into APOLLONIUS, I "admire the great man; and I may say the same of GREGORY of St. Vincent "and of HUYGENS; but who can help complaining of the tediousness of their "demonstrations? They seem to me in many cases like a man who would "reject the Indian symbols of number, and perform all the arithmetical "operations in words at length. This might be done; but to what good "purpose? When I say to what purpose, I mean geometrical purpose. Other-"wise I much approve of the elements being demonstrated in words at "length, because of their logical use. For in the affairs of life we have no "symbols, but common words; and it is of service to habituate the mind, to "accurate and distinct expressions, and even to formal syllogisms."

In the same letter after, mentioning the Propositions of the *Data* from the 27th to the 62d, as very easy, he adds: "In these Propositions I see "little or no room for algebra; and indeed, in questions of position, it often "comes in aukwardly enough. Dr. WALLIS, one of the great patrons of "this method, owns it. It is therefore a proper enquiry to determine, if "possible, when one method is to be used, and when the other. You seem to "promise something of this kind in the preface to your *Conics*. I wish you "could find resolution to execute what I am persuaded is almost ready in your "manuscripts. THOMAS SIMPSON, in his *Exercises*, says, that sometimes "the algebraical, sometimes the geometric method is preferable. I believe it. "But it seems to me that notwithstanding the praises bestowed by Sir ISAAC "NEWTON and his followers on the method of the ancients, his own *Principia* "are but algebra disguised, as MACHIN used to say. Though at the same

"time he owned that he thought Sir ISAAC's taste of demonstration preferable "to HUYGENS's. I own myself of his opinion, as the only difficulty attending "Sir ISAAC's arguments, are the steps suppressed for the sake of brevity."

I shall add an extract from another letter of Mr. SCOTT's, dated 1st May 1764, as it is connected with the subject of the preceding.

When mentioning Dr. SIMSON's edition of the *Data*, he adds, "I think "that such as would form themselves upon the model of the ancients, will "do well to study the *Data*. I even wish that I had been initiated in that way. "But having once begun in the modern analysis, I have not had leisure to "strike into another road, especially, as, to confess the truth, it seems to me "much more tedious and more doubtful with respect to success. I mean "not that the method of the ancients leaves any doubt in the mind; but "that the success of attempting the solution of a difficult problem is more "doubtful in the way of the ancients, than in that of the moderns; although, "when the solution is once found, it be often true that demonstrations formed "according to the ancients are more agreeable, and more satisfactory to "the mind, than those derived from algebra. I say often, for it is far from "being always so. To me a demonstration derived from the consideration "of a multitude of ratios, and their compositions, and expressed in the "manner of the ancients, is so far from being clearer than the algebraical "method, that it seems vastly more difficult and obscure. And I would "appeal to any beginner, and ask him, whether the general algebraic solution "of the problem for determining the *Foci* of Lenses, or HUYGENS's method, "be most clear. I might bring many other instances, where Propositions "can be demonstrated in the same manner as EUCLID has done in his "first four books, which often reduce the truth of a Proposition to almost "intuitive evidence. I admit that it would be in vain to look out for any "assistance from algebra; and yet I think with Dr. BARROW, that most "of the Propositions of the second book are rather clearer when treated "algebraically, than in EUCLID's method. The perspicuity of the fourth "of that book has often been insisted on. It is very clear, undoubtedly; "but still our friend Dr. MATTHEW STEWART thinks that the original "demonstration was from the preceding Propositions, without a figure, "which is entirely analogous to the algebraic method. The fifth book is "not peculiar to geometry, but is equally applicable to quantity as it is to

"magnitude; nor does the doctrine of proportion receive any evidence from "its being expounded by lines. The symbols of algebra would, I think, be "of use here. In the sixth book there are many Propositions which may "be treated either geometrically or algebraically, without any preference of "advantage on either side: for wherever an analogy is used, it may as justly "be said to be algebraical, as it can be called geometrical. I may demonstrate "geometrically the 47. El. 1. from the doctrine of similar triangles, and I "may do the same algebraically. The two demonstrations differ only in stile; "and GUISNEE is mistaken, when he says this theorem cannot be demonstrated "algebraically. I readily admit that the theorems about the congruity and "similitude of triangles cannot be demonstrated by algebra, and that these "are the true geometrical elements; which being established, the other Pro-"positions in the *Elements* may be derived, either by the method of the "ancients, or by that of the moderns, equally well; at least, if the ancient "method has the advantage in easy questions, that of the moderns seems "of almost indispensible necessity in more difficult enquiries. I admit, however, "that many parts of your writings, and of Dr. MATTHEW STEWART's, might "furnish strong objections to what I advance, nor do I pretend to answer "such objections: all I say at present is, that I know a person very well versed "in both ancient and modern geometry, who seems diffident of his own "abilities to comprehend Dr. STEWART's determination of the Sun's distance. "He and I would, I believe, find the same difficulties in Dr. STEWART, "that BULLIALDUS did in ARCHIMEDES. And to conclude the subject, "I must confess that I cannot find that superiority of evidence in the method "of the ancients, that you and many other able geometers, in this island, "contend for. Their great genius and accuracy I do see, and I see sometimes "the precipitation and innacuracy of modern geometers. But these are the "faults of individuals, and not of the art."

In the same letter, after some remarks on the data not connected with the particular subject of the preceding extract, he adds, "In your notes you "bring two instances of the use of the data, and think that the construction "of the last problem could not be derived from an algebraical solution. "I find, indeed, that the algebraic solution is more complicated than one "would at first suspect, on reading the problem. Even in very small numbers, "the solution brings you to large ones. Thus, if $x:y::6:7$. also $y-1:z::3:4$

"and $xx + zz = 100$. I find $x = \frac{168}{277} \pm \frac{\sqrt{28224 + 2203812}}{277}$ and

" $\frac{\sqrt{2232036}}{277} = \frac{1494}{277}$, therefore $x = \frac{168}{277} \pm \frac{1494}{277} = \frac{1662}{277} = 6$.

" and $x = -\frac{1326}{277}$, therefore $y = 7$, and $-\frac{1547}{277}$

Alfo $z = 8$, and $-\frac{2432}{277}$, and both roots anfwer the

" queftion.

" In the general folution, fuppofing

$$x : y :: a : b$$
$$y - a : z :: a : c$$
$$xx + z^2 = d^2$$

" I find $\frac{bc}{a^2} x - c = z$, then taking $a : b :: c : f = \frac{bc}{a}$ and

" $a : \sqrt{aa + ff} :: \sqrt{aa + ff} : h = \frac{aa + ff}{a}$. Alfo $b : c :: f : \frac{cf}{b} = p$. I have

" $xx - 2px = \frac{a}{h}(dd - cc)$ and the conftruction derived by this method is not
" fo fimple as yours. But the equation $xx + zz = dd$ being obvioufly a locus
" ad circulum, and the equation $\frac{bc}{aa} x - c = z$, a locus ad rectam, I find an
" eafy conftruction by combining them. For taking $a : b :: c : f = \frac{bc}{a}$ I have
" $\frac{f}{a} x - c = z$, which is very eafily conftructed; and then defcribing a circle
" from the origin of the abfciffa x with the Radius d, I have the other Locus,
" the interfection of which with the former determines the value of x. This
" is not lefs fimple than yours; and you have alfo taken the advantage of the
" Locus ad circulum.——Hitherto there appears not to me any advantage in
" the ancient method. Nor do I fee any from the firft of the inftances given
" at the end of your *Conics*. It is true your folution is more fimple than
" NEWTON's, but your inveftigation is not peculiar to ancient geometry. For
" what fhould hinder an algebraift from fuppofing the angle BCD (in your figure)
" equal to BAC, and then calling $BD = x$, $DC = y$, and $AB = a$, we have by
" the fimilar triangles, and by multiplying the extremes and means of the
" analogy, $ax + x^2 = y^2$, which is a Locus ad hyperbolam æquilateram, very

"easily constructed. If Sir ISAAC's investigation be more complex, it is not "because he made use of algebra, but because he did not make a convenient "election of terms. Now this election cannot be taught precisely, though "Sir ISAAC himself has given many excellent observations relating thereto. "It is a sagacity and habit to be acquired by the study of good models; and in "this respect there can be no doubt but the study of the ancients may be of "great use, even to an algebraist. I believe Mr. D'ALAMBERT speaks truly, "when he says, that the study of geometry is perhaps too much neglected."

"After some other observations, he adds. "To conclude, I long much to "see a treatise of which you gave us hopes in the preface to your *Conics*, when "you say *in quibus autem differat analysis geometrica ab ea quæ calculo "instituitur algebraico, atque ubi hæc aut illa sit usurpanda alias disserendum.* "Such a treatise is much wanted; and I am persuaded that you must have it "almost ready for the press among your papers, so that the labour of publishing "would not be great."

These letters, it is obvious, were written by Mr. SCOTT without any intermediate letter from Dr. SIMSON. I shall subjoin the material parts of the Doctor's two answers, which he dispatched separately, though he mentions that he had received the second before he had finished his answer to the first.

Dr. SIMSON's first letter, dispatched July 27, 1764.

"I am now, by the weakness of my memory and of my eyes, in no good "condition to write letters of any length; but yet I shall try to answer the "several particulars you mention, to the best of my ability. I am glad the "Problems have come to your hands. You perhaps have not yet leisure to "read them, else I would have expected your opinion about them. Your "censures and corrections I value more than commendations. In the preface "to the *Data* it is said, that they are of the most general and necessary use "in the solution of problems of every kind; and whoever tries to investigate "the solutions of problems geometrically, will soon find this to be true; for "the analysis of a problem requires that consequences be drawn from the "things that are given, until the thing that is sought be shewn to be given. "Now supposing that the book of the *Data* were not extant, these consequences "must be found out and demonstrated from the things given; which, in most

" cafes, would take no little time and pains. But now having that book, the " Propofitions of it need only be cited, or fuppofed to be known; for though " the ancients did not cite the particular Propofitions either of EUCLID's " *Elements*, or of his *Data*, yet they fuppofed them to be known, as is evident " from the folutions of problems given by PAPPUS, in which he always makes " ufe of the Data as being known to every geometer. The Elements are " neceffary in every geometric reafoning, whether in demonftrating theorems " or in refolving problems; and in the analyfis of problems the Data are of " as general and neceffary ufe.

" The demonftration of the 17th Dat. which you gave in fymbols, is very " right; the fhortnefs of the expreffion by fymbols is no doubt of ufe; but " then there is greater hazard of committing miftakes by ufing them, than " when the words at length are made ufe of; and I believe that Dr. BARROW's " miftake in this Propofition has been owing to this fhort-hand writing, which " coft more time to underftand, than if he had ufed the ordinary way. As A " is with him the given magnitude, and $\frac{B}{C}$ the given ratio, it is plain that " A+B is the firft of the magnitudes mentioned in the enunciation; from " which taking A, the remainder B has to C, the fecond magnitude, a given " ratio; and the thing he fhould have demonftrated is, that A+B is major " A+B+C data quam in ratione, which he has not done, but only fhewed " that A+B is major B+C *d. q.* in *r.* Many fuch miftakes are obfervable " in problems done algebraically; and in the 31 Prop. lib. i. of the *Loci plani* " p. 96, I have taken notice of a remarkable one in SCHOOTEN's *Exer-* " *citationes*, p. 249. The given magnitude AE (in your notation $a-\frac{a^2}{a+b}$,) which " is to be taken from AB (or x) is eafily feen to be the excefs of AD (or a) " above ED, which is the 4th proportional to DC, DB and AD the given " magnitude; and if the given ratio of DB to BC be a to b, as you make it, " then the ratio of DC to DB will be that of $a+b$ to a.

" You feem to think, that wherever magnitudes and their ratios are con- " fidered, that the algebraical method is better than that of the ancients in " refpect of facility, though not of accuracy. But what if the conftruction " deduced from the firft method, be nothing fo good, either in refpect of " fhortnefs or elegance, as that from the other? which is often the cafe.

"You may at leifure try the algebraic folution of Prob. 2. in pages 465 and "466 of the Englifh EUCLID, which will furnifh you with an inftance of this.

"The length of the demonftrations in the method of the ancients arifes, "for moft part, from the ftrict regard they had to accuracy; yet I do not "fay that none of them can be made fhorter; for fometimes, by making "ufe of a more proper medium of the demonftration, they can be fhortened: "an inftance of which you have in the 20th Prop. b. ii. of APOLLONIUS's "*Conics*; which, by making ufe of the properties of the Affymptotes inftead of "thofe of the Diameters, is made fhorter in the 40th Prop. of b. iii. of the "book which I publifhed fometime ago. I expreffed myfelf indiftinctly "in the preface to that book, which has made fome think I defigned to "publifh fomething, to fhew when the ancient and when the modern method "ought to be ufed; but I meant to fay nothing more than that it was not "proper to be done in that preface."

"I cannot guefs on what account Mr. MACHIN, to whom you affent, thought "Sir ISAAC's tafte of demonftration preferable to that of Mr. HUYGENS. "Though I have the greateft veneration for Sir ISAAC's profound and penetrating "genius, and his moft valuable and ufeful difcoveries, yet I cannot but prefer "HUYGENS's elegance and perfpicuity, efpecially in his *Horologium oscilla-"torium*, to Sir ISAAC's; nor do I think the firft has fallen into fo many "miftakes as the other, in things purely mathematical."*

I fhall fubjoin fome extracts from Dr. SIMSON's letter in anfwer to Mr. SCOTT's 2d letter, of May 1, 1764.

"DEAR SIR,

"Your approbation of the edition of the *Data*, with which you begin your "fecond letter of May 1ft, gives me great fatisfaction. You obferve that the "demonftrations made by a multitude of ratios and their compofitions, ex-"preffed in the manner of the ancients, are more difficult and obfcure than "the algebraic method. I confefs a multitude of them makes a demonftra-"tion difficult in any of the methods, and I found it was not only difficult to "my fcholars to underftand the 20th Prop. of HUYGENS's *Dioptrics*, but "uneafy to me to prelect it to them, by reafon of the great number of ratios "that are confidered in the demonftration, where there is only a ftraight

* Sir ISAAC NEWTON himfelf entertained a high opinion of the elegance of HUYGENS's tafte in geometry. See Pemberton's View, Preface.

"line and its parts to aid the memory: and on this account I contrived a "demonſtration by help of a figure, by which the compoſition of ratios is "avoided, and nothing but the properties of parallel lines, and ſimilar "triangles, enter into; it which made it much eaſier both to my ſcholars and me."

"No doubt the fourth of the ſecond book of the *Elements* might have been "ſhewn from the third and the ſecond of it, and EUCLID, or even a far leſs able "geometer, could not but ſee it; but he preferred the way by figures in all the "demonſtrations of the ſecond book, as ſhewing the equality of the ſpaces "compared to the eye, which is more clear than when it is perceived by "the imagination. As to the fifth book, I do not ſee that the demonſtrations "in it can receive any help from algebra; and the ſtraight lines made uſe of "in it make the demonſtrations clearer and eaſier, than they would be without "them. I do not underſtand your meaning, when you ſay any analogy "may be called algebraical as well as geometrical. The expreſſing lines by "a ſingle letter does not make analogy or any thing elſe algebraical, any "more than when they are expreſſed by the two letters at their ends, nor do I "think any thing can be called algebraical, where no operation peculiar to "algebra is made uſe of. I ſhould be glad to ſee an algebraical demonſtration "of ſome propoſition in the 5th book, *ex. gr.* of the 17th, by which perhaps "I might underſtand your meaning. The 47th Prop. of 1ſt B. cannot be done "by ſimilar triangles in the place where it now ſtands; and when it is done "by them, I do not ſee that the demonſtration has the leaſt dependence "on any thing in algebra. You think algebra almoſt indiſpenſibly neceſſary "in difficult enquiries: I have, on the contrary, been obliged to make uſe of the "ancient method in many problems, in which I could not find that algebra was "of any uſe to me; ſuch were the two problems, the ſolutions of which I ſent "ſome time ago; and ſo are many others, of which the following is one. *Three "points and three straight lines being given in position, it is required to place "a triangle so that the three angular points may respectively be on the three "given straight lines, and the three sides pass through the three given points.*"*

"The ſolution which you give of the laſt problem at the end of the *Data*, "by help of two *Loci*, is very good; but I am not yet perſuaded, that the con-

* The enunciation in this letter has reference to a figure which is here omitted. It is alſo to be remarked that in the *Opera Reliqua*, p. 388, is a ſolution of a caſe of this problem, viz. when the three given points are in one ſtraight line.

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" struction arising from it, or the demonstration of it, will be quite so simple " as those given in that place: if you had leisure to write them out, without " making use of any algebraical notation or operation, they might then be " better compared. You think I have taken advantage of the *Locus ad Cir-* " *culum*, but I assure you it never came into my mind: the 34th Prop. in " the *Data*, as you will see by looking to it, does indeed require the description " of a circle, by which the position of the straight line AD (in Prob. p. 465,) " is found.

" It would take up too much time, and writing too much for my eyes, to " shew the advantage of the ancient method above the algebraic, and how the " precepts given in this last method, particularly in the *Arithmetica Universalis*, " lead those who observe them, from the right solution of geometrical " problems, into such as are quite out of the natural method; many instances " of this occur in that book, among which is that problem which is the " first at the end of the second edition of the *Conics*. You say the solution " there given is not peculiar to the ancient geometry, and you add, 'what should " hinder an algebraist to suppose the angle BCD in my figure equal to BAC " &c.?' True, an algebraist may give a geometrical solution of a problem, but it is " not therefore an algebraic solution. The solution you give in the algebraic " notation depends nothing on algebra, for the *Locus* $ax+x^2=y^2$ flows from " one of the first properties of the hyperbola demonstrated in the *Conics*, " and is the very same which is made use of in the 1st Prob. of the appendix. " It may be said that algebraists are blameable, not so much in making a wrong " choice of a medium, as for hindering themselves from making any choice " not pointed out by their rules; one of which almost constantly made use " of by them is, to let fall a perpendicular, that by help of the 47. 1st Elem. " they may get an equation, which otherwise they could not so easily obtain.*

* An observation of DES CARTES tends to confirm some of Dr. SIMSON's remarks in this letter; for which see BRANKER's Algebra, p. 65, (1668.) DES CARTES, in a letter " not yet printed, writes thus: *In searching the solution of geometrical questions, I always* " *make use of lines parallel and perpendicular as much as is possible; and I consider no other* " *theorems but these two.* (The sides of like triangles have like proportions,) *and* (in " rectangle triangles the square of the greatest side is equal to the square of the other two " sides,) *and I am not afraid to suppose many unknown quantities, that I may reduce the* " *proposed question to such terms, as that it depends on no other theorems but these two.*" This letter of DES CARTES, however, was afterwards printed, Amst. 1683, Epist. 72, part iii.

"I forgot, when mentioning the laſt problem, at the end of the *Data*, to "obſerve that in the equation you firſt bring it to, you give the value of "what is called the negative root, viz. $x = -\frac{1326}{277}$.† It is one of the abſur-"dities introduced into algebra in the laſt age, to ſuppoſe every equation has "as many roots as there are units in the index of its higheſt power, and "conſequently that every quadratic equation has two: but the contrary, in "the preſent caſe, can eaſily be ſhewn. Your equation is of the ſame form "with this $x^2-4x=12$, the root of which you know is $x=\sqrt{12+4}+2$ or 6. "And that this equation can have no other root, *i. e.* that there can be "no number but 6, from the ſquare of which taking the quadruple of the "number, the remainder will be 12, appears thus. Suppoſe it poſſible, that "y is alſo the root of the equation, or that $yy-4y=12$. Then y muſt be "either greater or leſs than 6 or x; (if any deny this axiom, I will reaſon "no further with him on this matter;) ſuppoſe that it is greater, $y-4$ is greater "than $x-4$, and therefore the product of y by $y-4$ is greater than the product "of x by $x-4$; that is, $yy-4y$ is greater than $xx-4x$, and conſequently than "12. Therefore $yy-4y$ is not equal to 12; that is, y is not the root of the "equation. The ſame thing follows, if y be ſaid to be leſs than 6.

"*The passage you have set down from the preface to the Conics, might, "I confess, make any one think that I designed to have published something "on the subject there mentioned; but indeed, as I wrote in my last, I blun-"dered in the expression, and have no papers, and never wrote any about "that matter. R. S.*"*

page 296. The ſentences ſucceeding the foregoing extract give additional explanations of his opinion, to which the reader is referred. I may alſo remark, that in this letter, (Ep. 72.) and in the following, (Epiſt. 73.) ſome difficulties are mentioned in the algebraical ſolution of the caſe of the *tactions*, in which three circles are given.

† Dr. SIMSON's notions about negative quantities in algebra have been frequently mentioned. His comment on the negative root of a quadratic equation, mentioned in Mr. SCOT's letter, though not connected with the other matters in this correſpondence, is retained as a characteriſtic ſpecimen, in his own words, of the Doctor's manner of treating that ſubject. He made no ſecret of his opinions reſpecting it: they were well known to all his contemporaries; and whatever objections may be made againſt them, he remained firm in maintaining them.

* This letter was diſpatched to Mr. SCOT, 13th Auguſt 1764; and no trace of correſpondence between theſe gentlemen, of a ſubſequent date, remains.

With respect to this correspondence mathematical readers will form their own opinion; but it is proper to remark, that Dr. SIMSON, at the time of writing his letters, was in his 77th year. I venture to add a few remarks, which, to those who have not considered the subject particularly, may be acceptable.

In all those geometrical investigations, in which the chain of argument can be rendered by the ancient analysis both intelligible to the understanding, and pleasing to the imagination, Dr. SIMSON's reasoning in its favour will not be objected to. It must be admitted also, as he states, that unless operations peculiar to algebra be introduced, an investigation, though clothed in the characters of algebra, may yet be properly geometrical. He gives an example of the solution of a problem by the combination of two local equations; and such may always be regarded as purely geometrical, when the *Loci* employed have been demonstrated in known geometrical works, and are easily deducible from the *Data* in the problem without any algebraical operation. But even when an algebraical process is requisite for investigating the *Local Equations*; or when, in the investigation of a geometrical problem by algebra, some inferences may be more immediately drawn by means of known geometrical propositions than by the operations of that art; the algebraical solutions may by such aids be much abridged, and often rendered much more elegant.

It is further to be remarked, that when geometry comes to be employed in practical mensuration, or in physical enquiries, the most obvious and most expeditious methods of investigation, whether geometrical or algebraical, will usually be preferred. It is well known that in such applications, in general it is useful, and often absolutely necessary, to express conclusions (in whatever way they may be obtained) arithmetically; and hence it is frequently expedient to investigate by algebra such conclusions, though they might without difficulty be obtained by geometrical reasoning, as they must ultimately assume the arithmetical form in the practical uses of them. I must also observe, that Mr. SCOTT's remark respecting the combination of ratios is perhaps not fully answered by Dr. SIMSON; and that the remark may also in some cases be extended to any long series of steps, by the combination and separation of squares and rectangles arising from the segments of a straight line, by the use of the Propositions of the iid b. of EUCLID, or of others founded on that book. Though a demonstration may be regarded as strictly

geometrical, when ſuch ratios, and the combinations of them, are ſet down either in words at length, or in a kind of ſhort-hand, by uſing ſome algebraical characters; yet when the mind loſes the diſtinct perception of the particular geometrical magnitudes compared, the evidence is ſimilar in its impreſſion to that of algebraical reaſoning, in which the previous demonſtration of the rules employed is the ground of our aſſent to the truth of the concluſions, and not the immediate perception of the geometrical magnitudes and their relations.—An analyſis or demonſtration in which many combinations of ratios are employed, may generally indeed be much ſhortened by admitting multiplication, diviſion, or other operations peculiar to arithmetic or algebra; but the inveſtigation or demonſtration then becomes truly algebraical. It muſt however be admitted, in the particular caſe here ſuppoſed, that the evidence of the algebraical proceſs is not materially different, or rather that the impreſſion of that evidence on the mind, is not very different, from that of the geometrical method. The ſame obſervation may be extended alſo to the caſe, when many combinations of ſquares and rectangles from the ſegments of a ſtraight line neceſſarily make a principal part of a geometrical analyſis or compoſition.

Dr. Simson acknowledges the difficulty of purſuing ſuch long trains of reaſoning, by combining the ratios in the Propoſition of Huyghens particularly alluded to. He found it even expedient to contrive a new demonſtration by figures, which might facilitate his own labour in lecturing, and more effectually keep alive the attention of his ſcholars, by preſenting objects better calculated for that purpoſe, and alſo for aiding the memory in retaining the long ſeries of ſteps in that Propoſition. But in ſuch Propoſitions an algebraical deduction would in many caſes be not leſs ſatisfactory, and far more expeditious, than any other method.

It muſt be conceded, however, by the patrons of the algebraical analyſis, that the ancient method, where it can be applied, with very few exceptions, is more elegant, and gives a more immediate and more pleaſing conviction of the truth of a geometrical propoſition, than the other. It muſt be admitted, alſo, that there are many geometrical problems, even in the claſs denominated *plane*, of which the algebraical ſolution is far from being obvious, and is often more difficult than the geometrical; even from the complexity of the calculation, without regarding the inferiority of the conſtructions which

generally result from the solution of equations. Dr. SIMSON, in one of his letters to Mr. SCOTT, mentions an example; and others equally illustrative of the point might easily be pointed out.*

That many geometrical problems are more easily resolved by algebra than by the ancient analysis, there is no doubt; but when the object is to obtain a solution, in which rigid accuracy is combined with perspicuity in every step of both the analysis and demonstration, the ancient method is indisputably to be preferred. But the distinguishing value of the modern analysis, though not particularly considered in this correspondence, is the means which it affords of resolving geometrical problems, inaccessible to the ancient analysis in its present state; and of applying geometry to physical investigations, equally beyond the reach of that more elegant, though less powerful instrument. Such, indeed, has been the success of the modern analysis, in the vast enlargement of some important branches of science, and of the arts depending on them, that all reasonings on any theoretical imperfection in its principles, which does not affect the truth of the conclusions, is superseded by the wonderful extent and utility of its power and application.†

In a miscellaneous MS. volume of the year 1722 were found, annexed to the solution of a geometrical problem, some short observations by Dr. SIMSON on the ancient analysis; which, though apparently suggested merely by that solution, and written without any particular view, yet mark the accuracy of his notions respecting that analysis at this early period, before he had made much progress in the restoration of the lost analytical works of the ancient geometers. They may however, notwithstanding these circumstances, be considered as a suitable addition to the Doctor's letters on the subject in his old age, which have been

* The problem, (117 lib. vii. PAPPI) and the extensions of it, of which a short history is given in note E, may also be mentioned as examples.

† Such is the power of the modern analysis in many complex investigations, that an observation of WOLFIUS (tom. v. Math. Univ. p. 210) will not be considered as altogether extravagant, even by the rational admirers of ancient geometry. "Sane si "ARCHIMEDES et APOLLONIUS nostro ævo reviviscerent, in stuporem raperentur visis "inventis recentiorum, quæ per algebram fuerunt in apricum producta; neque unquam "sibi persuasissent patere ad talia mortalibus aditum." The original and powerful genius of ARCHIMEDES however, had his life been prolonged, might have anticipated many of the great discoveries of modern times; and have adorned them also with his elegant and accurate manner of demonstration.

given in this note. To prevent unneceſſary trouble, the particular references to the problem are omitted.

" Elegans eſt hoc exemplum veræ et genuinæ tam analyſios quam con-
" ſtructionis et demonſtrationis, methodo veterum inſtitutarum, quaſque algebra
" recentiorum nunquam præbuiſſet."

" In genere notandum eſt, analyſim eo puriorem et elegantiorem habendum
" eſſe, quo minus artificii in conſtructione analyſi præmiſſa invenitur; *i. e.* quo
" pauciores, eæque non niſi ob rationes obvias ducendæ lineæ, conſtructionem
" iſtam ingrediuntur.

" Analyſes enim hac methodo inſtitutæ præferendæ ſunt iis quæ, quamvis bre-
" viores eſſe poterint, tamen ope conſtructionis minus obviæ perficiuntur, *i. e.* hæ
" ipſæ analyſes alterâ analyſi indigent, ut ad iſtas conſtructiones perveniatur, quæ
" propterea minus perfectæ ſunt cenſendæ. Perfectiſſimæ enim ſunt analyſes quæ
" a datis procedunt ad iſta quæ immediate cum iis connectuntur, atque ita donec
" ad quæſitum perveniatur. Verum quidem eſt, eos qui ſagaci et acri pollent
" ingenio, poſſe per pauciores gradus ad quæſitum, quaſi ſaltando, pervenire, ſed
" hoc nullo modo poteſt fieri, niſi vel animo perceperint grados omiſſos, quod
" redit ad methodum deſcriptam, vel tentando inciderint in conſtructionem
" quandam, a datis ſatis remotam, id quod omni cura ab analyſta eſt vitandum.
" Cum nihil verâ et genuinâ analyſi magis diſtat, nihil magis abhorreat, quam
" tentandi methodus; hanc enim amovere et certiſſima via ad quæſitum per-
" ducere, præcipuus eſt analyſeos finis; quamvis fatendum, tentando et in
" tenebris quaſi palpitando, quæſitum poſſe inveniri, ſed hac methodo nunquam,
" niſi cogente demum, ob deficientem analyſim, neceſſitate procedendum. De
" compoſitionis et demonſtrationis natura, cum eæ ex analyſi ſemper efficiuntur,
" eodem modo judicandum.

" Porro animadvertendum eſt nihil in analyſi de quæſito vel ejus quavis affec-
" tione ſupponendum quod ex datis non clare fluat. *Ex gr.* In inveſtigatione
" loci alicujus non ſupponendum eſt locum eſſe *rectam* vel *circulum*, quoniam
" hoc eſt quod quæritur. Et hoc vitio laborat analyſis ex qua conſtructionem
" ſuam Prob. in Prop. 7, p. 25, lib. i. APOLLONII *de Locis Planis*, hauſiſſe
" videtur FERMATIUS, nam inveniendo duo puncta in quibus locus quæſitus
" ſecat datas poſitione rectas, et affirmando rectam, hæc duo jungentem, eſſe
" locum quæſitum, videtur eum etiam in analyſi ſuppoſuiſſe, locum quæſitum
" eſſe rectam.

"Notandum denique in difficilioribus problematis tractandis multum ad "folutionem conducere, fi a cafu aliquo fimpliciore inciperimus."

He proceeds to illuftrate this laft obfervation by a particular cafe, which is too long for infertion.

APPENDIX.

APPENDIX I.

Account of the Mathematical Collections of PAPPUS.†

§. I. *Two first Books of* PAPPUS.

PAPPUS, an eminent Mathematician of Alexandria, flouriſhed about the end of the fourth century.‡ In the very brief accounts which remain reſpecting him, he is mentioned as the author of ſeveral treatiſes all of which; except his *Mathematical Collections*, probably the moſt valuable of his writings, appear to have periſhed. This work, as the

† In Dr. HUTTON's learned work, the *Mathematical Dictionary*, there is an excellent abſtract of the contents of the *Mathematical Collections*, to which I at firſt intended to have referred. But on further conſideration, I found that, for the purpoſe of giving a ſatisfactory account of Dr. SIMSON's commentaries and corrections of that work, a more detailed ſtatement was neceſſary, as will readily appear to the intelligent reader.

‡ See SUIDAS; (Πάππος,) alſo GER. VOSSIUS *de Chronologia Mathematicorum*; and MONTUCLA tom. i. In SUIDAS ſeveral works of PAPPUS are mentioned; but neither his Collections, nor a treatiſe alluded to by himſelf in b. iv. of the Collections, (COMMAND. fol. 56, 1588.)

name imports is miscellaneous, and besides a variety of Propositions, both problems and theorems, contains some curious notices, not to be found elsewhere, of the history of mathematics and of mathematicians in his own and in preceding times. Its chief importance however is, as has already been mentioned, that from it alone we derive satisfactory information respecting the ancient geometrical analysis. In the preface to the seventh book, particularly, is given a description of the most valuable treatises in the collection, which in the celebrated school of Alexandria* got the title of τόπος ἀναλυομένος, and which was composed by some of the most eminent geometers of antiquity, for promoting the use of that analysis.

Of the eight books of the *Mathematical Collections*, the first and about one half of the second are presumed to be lost; the rest have reached the present time, though with many imperfections, and in some passages so mutilated, that the meaning cannot be certainly ascertained. The original Greek (except some short extracts) has never been printed; and the only translation of it, by COMMANDINE, was first published at Pisaurum in 1588; and another edition, with little variation or improvement, was printed in 1660 at Bologna.† This translation is

* It is remarked by VOSSIUS, in the before-mentioned work, "ab EUCLIDIS tempore usque ad tempora Saracenica, vix ullum invenire sit nobilem mathematicum, quin vel patria fuit ALEXANDRINUS, vel saltem Alexandriæ dederit operam MATHESI."

† Though the second edition is said to be corrected, yet this appears to be only in some press errors of the former edition, while new errors of the same kind are introduced: but there is no attempt to amend the obscure and deficient passages in that translation. From some convenience of the bookseller, the original edition succes-

accompanied with a commentary, often tedious, and in ſome places defective; but at the ſame time extremely valuable, from the explanation which it contains of ſome difficulties, and the correction of many errors in the MS. uſed by COMMANDINE, and which pervade all the MSS. of PAPPUS that have hitherto been examined. From COMMANDINE's manner of referring to the Greek, it appears that he had only one MS. for his guide. He died before the work had received his laſt corrections; and no account is given of the hiſtory or character of the MS. which he followed.† From a family diſpute between two ſons-in-law, the publication was ſuſpended for ſome time after his death; and at length, by the munificence of his patron the DUKE of URBINO, the tranſlation was printed, but confeſſedly without any correction whatever of the errors or omiſſions in the unfiniſhed work of COMMANDINE.‡

In this ſtate, however, it was a very intereſting communication to the mathematicians of that age, and excited much curioſity reſpecting ſome branches of the ancient geometry, which, before this publication, were little conſidered or known in modern Europe. Accordingly, not long after its appearance,

ſſively got three title-pages and dates; the firſt, Piſauri 1588; the ſecond, Venetiis 1589; and the third, Piſauri 1602.

† COMMANDINE in many hundred notes quotes the CODEX GRÆCUS. In a very few places indeed he does quote the GRÆCI CODICES; but this laſt expreſſion ſeems to have ariſen merely from inadvertence, as in no inſtance is there any compariſon of readings of different MSS. though, in a great many caſes, from the ſenſe, he corrects the blunders of the one manuſcript to which he uſually refers.

‡ See the Dedication, and alſo the addreſs to the reader, in which theſe circumſtances are mentioned; and it is alſo obſerved that COMMANDINE had made diligent ſearch for the two firſt books of PAPPUS, but without ſucceſs.

SNELLIUS, VIETA, MARINUS GHETALDUS, and FERMAT, attempted, from the defcription of the τόπος ἀναλυόμενος in the feventh book, to reftore feveral of the loft books of APOLLONIUS, which made a principal part of that collection. Afterwards SCHOOTEN, Dr. WALLIS, and fubfequently Dr. HALLEY, purfued the fame track; and thus a number of thefe ancient treatifes were reftored, with various fuccefs indeed, and with different characters of elegance and accuracy, as has already been obferved in the foregoing Memoir.

The two firft books of PAPPUS are not in COMMANDINE's tranflation, from their not being found in any of the MSS. to which he had accefs; and they are wanting alfo in feveral other MSS. remaining in various libraries of Europe.* In a MS. of PAPPUS, in the Savilian Library at Oxford, (No. 9,) there is preferved about one half of the fecond book, which was publifhed by Dr. WALLIS in 1688, with a Latin tranflation, and valuable notes, explanatory of the Greek arithmetic.

From this remaining fragment, it is reafonably conjectured by Dr. WALLIS, that thefe two books related folely to that arithmetic; and thence he infers that the lofs of them is not greatly to be lamented. We learn from the fragment, indeed, that APOLLONIUS had compofed a treatife on this fubject, to which PAPPUS frequently refers; and Dr. WALLIS derives from it fome information refpecting the notation and algorithm of the ancient arithmetic. From the defective mode of notation

* See at the end of this Appendix a fhort notice of the MSS. of PAPPUS, which are generally known to be preferved in the libraries of Europe. It may be obferved in this place, that fome of them, befides the Savilian MS., have the fragment of the fecond book publifhed by Dr. WALLIS.

among the Greeks, as compared with that of modern Europe, though there be much ingenuity in ſome of their methods, we need not be ſurpriſed at the great inferiority of their ſyſtem. Dr. WALLIS remarks, that the buſineſs of the ſecond book of PAPPUS appears to be nearly equivalent to what is now conſidered as a very ſimple propoſition, viz. that the multiplication of any numbers, all or any of which have cyphers annexed, may be performed by multiplying theſe numbers without the cyphers, and then adding all the cyphers to the product. The firſt book he with much probability conjectures to have been employed about the ſimple operations of the addition and ſubtraction of numbers.* And I need only further remark, that there is no appearance among Dr. SIMSON's papers of his having directed his attention to this fragment.

The ſucceeding five books of PAPPUS are geometrical, and the laſt is almoſt entirely mechanical. They are all of a miſcellaneous character; though in ſome of them there is a certain degree of method, and titles have been prefixed to them, alluding to the principal matters contained in the ſeveral books.† Theſe titles do not appear to have been in the MS. followed by COMMANDINE, as he does not mention them in his tranſlation; and probably, indeed, they did not belong to the original work, but were added by ſome commentator, perhaps of a much later age. Though I purpoſe to give a particular account of the contents of the ſeveral books, it may be proper

* See WALSII *Fragmentum Secundi Libri* PAPPI, aud the *Epilogus*.

† The MS. BOLL. has thoſe titles, from which they are copied in this Appendix. The tmo Savilian MSS. at Oxford have them, and likewiſe the Pariſian MSS.

first to mention the distinctions of geometrical propositions (especially of problems) assumed by the ancients, as they are stated by PAPPUS in two passages of his work, expressed nearly in the same words.‡

COMMANDINE's translation of these passages is given in Appendix II. and the Greek not having been printed, is added from the two Savilian MSS.§ and the MS. BULL. for the satisfaction of those who may wish to consider them accurately. From these extracts it appears that it was the difficulty, or rather the impossibility, of resolving some problems by the circle and straight line, which suggested the investigation of other curve lines, by the description of which the solution of such problems might be accomplished. The doubling of the cube,* and the trisection of an arch of a circle, were two celebrated problems, which exercised the ingenuity of the more ancient geometers, but which were found not to be resolvable by plane geometry. From the very brief accounts which remain of these speculations, it appears that the first attempt of producing new curves, which might be employed in geometrical science, was from the section of a solid by a plane; and the only solids considered in the early state of the science, which, by such a

‡ The first passage is in the third book, fol. 4. b. in COMMANDINE's PAPPUS, (1588,) and the other in the fourth book, fol. 60.

§ Accurate copies of these passages from the two Savilian MSS. were procured for me, in the most obliging manner, by the Rev. Dr. ROBERTSON, Savilian Professor of Geometry; who also gave me every facility for consulting these MSS. and other books in the very curious collection placed under his care. I met with the same obliging attention, when consulting some MSS. and books in the Bodleian Library.

* Usually called the Deliacal Problem, from the well-known tradition about doubling the cubical altar at Delos.

ſection could produce curves different from the circle, were the cylinder and cone. But as the ſections of the latter comprehended the curves reſulting from the ſections of the former, the three new curves, ariſing from the different poſſible ſections of the cone by a plane, obtained the name of Conic Sections. By theſe curves the two before-mentioned problems were eaſily reſolved; and from this origin, all problems requiring for their ſolution the deſcription of one or more of them, were called ſolid, though they had no other relation to ſolid figures.

Some other curves were alſo invented by ingenious men of thoſe times for the ſame purpoſe; but the Ancients did not purſue this branch of geometry, and conſidered only a ſmall number of ſuch lines, without having had any notion of the unbounded number which modern ſpeculations have brought into notice; and therefore, without propoſing any principle of ſyſtematic arrangement. All curves, beſides the Conic Sections, according to the account of PAPPUS, in the paſſages alluded to, were by the ancients denominated *Lines*, and problems reſolvable only by ſuch curves were called *Linear* Problems. All geometrical problems therefore, among them, were divided into three claſſes, *plane*, *ſolid*, and *linear*; and theorems were ſometimes diſtinguiſhed in the ſame manner, as they had reference to thoſe three claſſes of lines. The ſuperior lines treated of by PAPPUS, and other ancient writers, were the *conchoid*, the *ciſſoid*, the *ſpiral*, and the *quadratrix*; and a few others are ſlightly alluded to.

In the ſolution of the two celebrated problems, which are ſuppoſed to have given riſe to the inveſtigation of new curves, ſome of theſe laſt-mentioned curves have been employed, as

well as the Conic Sections. And we are informed by PAPPUS, that the difficulty of describing the Conic Sections with mechanical accuracy led some of the ancient geometers to employ those higher curves, the description of which was found to be more easy. The conchoid in particular was used for finding between two given straight lines two mean proportionals, from which the doubling the cube was an obvious inference; and the trisection of an arch of the circle was accomplished also by the same curve, and likewise by the spiral and quadratrix.* From PAPPUS it appears, however, that the early Mathematicians had at first some reluctance in admitting either the Conic Sections or superior curves in the solution of problems, considering them as not strictly geometrical;† but afterwards these lines became objects of much curious investigation, even among the ancients; and in modern times ultimately were of the most extensive utility, both in abstract and in physical science.

The above-mentioned passages of PAPPUS naturally suggest an observation which may be stated in this place; that expressions occur on this subject, in the writings both of ancient and modern geometers, in which there seems to be some want of that precision and consistency which properly belong to the language of mathematical enquiry. The description of any geometrical line from the data by which it is defined, must always be assumed as possible, and is admitted as the legitimate

* These problems are resolved in that manner by PAPPUS, in b. iii. and iv.

† Even some of the more early of the Mathematicians, reckoned modern, entertained a similar notion; particularly VIETA.

means of a geometrical conſtruction: it is therefore properly regarded as a *postulate.* Thus the deſcription of a ſtraight line and of a circle are the poſtulates of plane geometry, aſſumed by EUCLID. The deſcription of the three *Conic Sections,* according to the definitions of them, muſt alſo be regarded as poſtulates; and though not formally ſtated like thoſe of EUCLID, are in truth admitted as ſuch by APOLLONIUS,† and all other writers on this branch of geometry. The ſame principle muſt be extended to all ſuperior lines.

It is true, however, that the properties of ſuch ſuperior lines may be treated of, and the deſcription of them may be aſſumed in the ſolution of problems, without an actual delineation of them. A plauſible though imperfect repreſentation of geometrical lines is indeed extremely uſeful in aſſiſting the imagination, when we are employed in the inveſtigation of their affections; but the degree of accuracy with which they are exhibited is of no importance to the truth of the reaſoning, or to the ſatisfaction with which it is perceived. For it muſt be obſerved, that no lines whatever, not even the ſtraight line or circle, can be truly repreſented to the ſenſes according to the ſtrict mathematical definitions;‡ but this by no means affects the theoretical concluſions

† See Prop. 52, 53, 54, APOLLON. b. i. in which the deſcription of the three ſections is aſſumed preciſely according to the definitions given of them by APOLLONIUS.

‡ CARTESII *Geometria,* lib. ii. at the beginning. It may alſo be remarked, that even in the *Elements* of EUCLID, particularly in the xith and xiith books, certain conſtructions are aſſumed, which, though perfectly conſiſtent with the rigour of geometrical demonſtration, would in mechanical practice be extremely difficult. But they muſt be conſidered as poſtulates, though not ſtated to be ſuch by EUCLID.

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which are logically deduced from ſuch definitions. It is only when geometry is applied to practice, either in menſuration, or in the arts connected with geometrical principles, that accuracy of delineation becomes important.

Among the ancients, the deſcription of curve lines with a certain degree of eaſe and accuracy was important; as we have reaſon to believe that ſuch deſcriptions were often uſed by them in practical applications of geometry to menſuration and mechanics. This the limited nature of their ſyſtem of computation rendered particularly expedient; and influenced by this conſideration, we may preſume, the Greek geometers, in the ſolution of problems reſolvable by the *Conic Sections*, ſometimes employed ſuperior curves, on account of the greater facility of deſcription; of which there are examples in the *Collections* of Pappus.* In recent times, however, the power of the modern analyſis, eſpecially with the aid of trigonometry, renders the accurate delineation of curves ſeldom neceſſary; and in the few caſes where it may be uſefully employed, thoſe curves ought no doubt to be aſſumed, the deſcription of which, in the exiſting ſtate of mechanical arts, moſt conveniently enſures the required degree of exactneſs.

But in a ſcientific conſideration of the ſubject, it is a principle admitted ultimately, both by the ancient and modern geometers, that the proper ſolutions of problems

* Pappus enumerates ſeveral ſolutions of the Deliacal problem, ſome of them purely mechanical, and one of them by the conchoid; a curve of a claſs ſuperior to the *Conic Sections*. Sir Isaac Newton even admits, that in certain caſes the conchoid may be preferred to the *Conic Sections*, on account of the ſimplicity of deſcription. *Arith. Univerſalis, Appendix, conſtructio Æquationum linearis.*

muſt be effected by the deſcription of the loweſt claſs of lines by which they are practicable; and that though conſtructions of problems by curves ſuperior to thoſe by which they are reſolvable, may be demonſtrably true, and in particular caſes, may be practically uſeful, yet in theory are to be rejected as irregular.

§. II. *The Third Book of* PAPPUS.

THIS book, as the title imports, contains geometrical problems, both plane and ſolid.*

The problems conſidered in this book are four: firſt, the duplication of the cube, or what is equivalent, the finding two mean proportionals between two given ſtraight lines, which is ſolid: ſecond, a problem reſpecting the mediatates: third, a ſort of paradoxical problem, to place two ſtraight lines, drawn from two points in one ſide of a triangle, to a point within it which may be greater than the other two ſides: fourth, to deſcribe the five regular ſolids in a ſphere. Theſe three are conſidered as plane.

The firſt problem, which was an object of much conſideration among the ancients, is reſolved by PAPPUS in ſeveral of the methods then known; but he begins with a refutation of an unſcientific attempt to reſolve it by plane geometry, which ſeems, in his time, to have excited ſome attention. The

* The title in MS. BULL. is Πάππου Ἀλεξανδρέως συναγωγῶν† Γ, περιέχει δὲ προβλήματα γεωμετρικὰ ἐπίπεδα καὶ στερεά.

† The transcript from the Parisian MS. in the Savilian library has συναγωγῶν μαθηματικῶν.

examination of it, which had been solicited from PAPPUS by the author and his friends, requires a long train of reasoning, from the complex construction which had been proposed by this inaccurate geometer; and some things, assumed by PAPPUS in this argument, are demonstrated in four propositions annexed to it. Both the text and the diagram have been injured by transcribers; but COMMANDINE's translation is much amended by Dr. SIMSON, and a few of his corrections, indeed, are confirmed by the MS. BULL. and several in the figure are from the Parisian MSS. communicated by Dr. MOOR. Before giving the particular solutions of the Deliacal problem, PAPPUS premises a statement of the ancient distinction of lines, already explained. In this account he mentions the solution of the Deliacal problem by means of the *Conic Sections*, without however giving a detail of that solution; but he explains several others, viz. three mechanical, by ERATOSTHENES, HERO, and PAPPUS himself; and a fourth of NICOMEDES, by the description of another curve, supposed to be invented by him, and called the *Conchoid*. EUTOCIUS, in his commentary on ARCHIMEDES (Prop. 2. lib. ii. *de Sphæra et Cylindro*) gives several other solutions, (ten in all,) and particularly two by MENECHMUS by the intersection of two *Conic Sections*.—It may be observed also, that in some of the MSS. of PAPPUS, at the end of the third book are placed some variations of the mechanical solutions of this problem, not taken notice of in COMMANDINE's edition.†

† In Prop. 24. lib. iv. The solution of this problem by the conchoid is repeated; and the solution in PAPPUS's method in this book is also repeated in lib. viii. Prop. 11.

The ſecond problem reſpecting the *medietates* is branched out into twenty-one Propoſitions, with ſeveral definitions and explanations. He diſtinguiſhes three ſpecies, and when three lines (or three numbers) are either arithmetical, geometrical, or harmonical proportionals, according to the common definitions of theſe terms, they were anciently called *medietates:* while the term *analogy* was uſually applied only to the geometrical proportionals; ſo that ſtrictly an *analogy* is a *medietas*, but not converſely.

The principal problem‡ which he propoſes, is to place the three medietates in a ſemicircle. The reſolution of this by PAPPUS is in Prop. 16th, but he begins with animadverting on a geometer of his time for his manner of treating the problem, though unjuſtly, as Dr. SIMSON obſerves in a note on this paſſage.§

Before this 16th Prop. however are ten problems according to COMMANDINE's enumeration, reſpecting the three medietates. But after that Propoſition he obſerves, that NICOMACHUS the Pythagorean, and others, had treated not only of theſe original medietates, but alſo of three* others which were added

‡ Called the ſecond: (The Deliacal problem being the firſt.) It is propoſed in fol. 7 b. COMMAND. PAPPI. And in the ſame page is the definition here referred to.

§ Dr. SIMSON quotes VINCENTIO VIVIANI *de Locis ſolidis*, lib. iii. page 90, where there are ſome Propoſitions concerning the medietates, and alſo a remark of this miſtake of PAPPUS. I may add, that BLONDEL, in his *Reſolution des 4 Problemes d'Architecture*, makes the ſame corrections of PAPPUS's cenſure; and adds, that the ſolution by PAPPUS (Prop. 16.) is defective. But his objection to that Prop. is rather verbal than geometrical. See BLONDEL, p. 36. 37.

* PROCLUS, in his *Commentary* on EUCLID, (p. 19, HERVAGII,) ſtates that EUDOXUS of Cnidus, about the time of PLATO, added three analogies (τρεις ἀναλογιας) to the three then known, and the former probably were the medietates alluded to in this place by PAPPUS.

among the ancients, and four more among the moderns, making ten altogether, which he defines and illuſtrates by a number of Propoſitions to the 27th incluſive. Theſe, particularly the ancient medietates, were much conſidered in the Platonic School, in which probably they originated, and to which PAPPUS belonged. ERATOSTHENES compoſed two books on the medietates, (probably the *Loci ad medietates*) and though they were part of the *τόπος ἀναλυόμενος*, PAPPUS gives no deſcription of them, and has preſerved no Lemmata connected with them.† This may be conſidered as a preſumption of his opinion that in his time they were not regarded as important; and he further remarks of the more ancient medietates, that they were uſeful, chiefly in explaining alluſions to them in ancient writings; and probably this was the principal object of the details which he has given reſpecting them.

The general nature of the third problem has been mentioned, and it is expanded into ſixteen Propoſitions. It is a ſort of paradox, and he refers to a treatiſe of ERYCEMUS on Paradoxes, which, in the time of PAPPUS, was generally circulated, and probably in ſome eſtimation. Some of the Propoſitions are curious, but in the preſent ſtate of the ſcience they cannot be conſidered important, and are intereſting only as examples of the geometrical ſpeculations of former times, (Prop 28—43.)

† From the obſervation of PAPPUS at the end of his deſcription of the *Inclinations* of APOLLONIUS, it would ſeem that theſe *Loci ad medietates* were *plane*, and, in the order of the *τόπων ἀναλυομένων*, placed laſt.

The fourth and laſt general problem of this book is to inſcribe in a ſphere the five regular ſolids; and as theſe ſolids were objects of much ſpeculation in the Platonic ſchool, they were naturally attended to by PAPPUS, who belonged to it. This problem occupies the remainder of this book in the laſt ſixteen Propoſitions, of which eleven are preliminary to the other five; and theſe laſt contain the ſolution of the problem reſpecting the before-mentioned five regular ſolids.

§. III. *Of the Fourth Book.*

THE fourth book is miſcellaneous, containing theorems of the three ancient claſſes of Propoſitions formerly mentioned; plane, ſolid, and linear.* The firſt may be remarked as an extenſion of the 47. 1 Elem.; and the 4th is a curious theorem, with an analyſis and compoſition, which was rendered more general by Dr. M. STEWART, in a paper publiſhed in the firſt volume of the Edinburgh *Phyſical Eſſays*;§ and ſome Propoſitions alſo are added by Dr. SIMSON, in his notes.

* The title in MS. BULL. is as follows: "Πάππου συναγωγῆς δʹ ὅπερ ἐστιν ἀθηρῶν‡ θεωρημάτων ἐπιπέδων, καὶ στερεῶν καὶ γραμμικῶν."

‡ Probably it ought to be ἀνθηρῶν, as it is in the title of the eighth book.

§ In Dr. STEWART's Paper, in that volume, are ſeveral Propoſitions connected with this propoſition of PAPPUS, and which are analyſed and demonſtrated in the ancient manner. A ſcholium, containing ſome conic theorems without demonſtrations, is added; and at the end of it is a theorem, alſo without demonſtration, which is truly a Poriſm, and is analyſed and demonſtrated by Dr. SIMSON, Prop. 91 of his treatiſe. Dr. STEWART, from delicacy no doubt to Dr. SIMSON, who wiſhed to keep his expo-

The 10th is the cafe of one of the problems of the *Tactions* of APOLLONIUS, in which three circles touching each other are given, to which the three preceding Propofitions are preliminary; all however requiring the corrections remarked by Dr. SIMSON.†

Before the 13th Prop. a property of the figure called *Arbelon*‡ is ftated, as an ancient Propofition generally known, and of which the demonftration is in the 16th, the intermediate Propofitions being neceffary to it.

At the 19th Prop. he gives the definition of the Spiral of CONON, and demonftrates fome of its principal properties.‖ This is the curve fully treated of by ARCHIMEDES, and in a fubfequent part of the book it is employed in the folution of fome problems. PAPPUS (Prop. 24) repeats the definition of the

fition of the Porifms fecret till he fhould publifh his treatife, ftates the Propofition as a theorem, though he was well acquainted with Dr. SIMSON's difcovery, and had inveftigated many curious Porifms.——See *Edinburgh Phyfical Essays*, vol. i. p. 141—172.

† There are fome amendments on the 12th, neceffary to make the demonftration complete.

‡ The Arbelon (ἄρβηλον) is mentioned in the *Lemmata* of ARCHIMEDES, (Prop. 4, 5, 6;) and it is remarkable, that though PAPPUS in his *Collections*, and particularly in this book, often quotes ARCHIMEDES, there is no allufion to the *Lemmata* in his long and curious difcuffion of the properties of the Arbelon. There is alfo a lemma of COMMANDINE's, for demonftrating a Propofition of PAPPUS on this fubject, (Prop. 14,) which is the firft Propofition of the *Lemmata* of ARCHIMEDES, and which COMMANDINE afterwards (fol. 52, b.) afferts to be compofed by himfelf. From this it may be inferred, that COMMANDINE certainly had never feen the *Lemmata* of ARCHIMEDES, and moft probably neither had PAPPUS. The *Lemmata* have never been found in Greek, and have by feveral learned men been fuppofed not to be the work of ARCHIMEDES; and the circumftance now mentioned favours that fuppofition.

‖ In Prop. 21, refpecting the Spiral, the demonftration is conducted on principles like the method of Indivifibles of CAVALLERIUS.

conchoid of NICOMEDES, and the ſolution of the Deliacal Problem by it, as in the preceding book: he afterwards employs this curve for reſolving another celebrated problem of antiquity, the triſection of an arch of a circle, which was found impracticable by plane geometry; and which, with the Deliacal Problem, rouſed the efforts of mathematicians to inveſtigate new curves for reſolving them.

He proceeds to explain the origin and properties of the quadratrix, (τετραγωνίζουσα,) aſſumed by DENOSTRATUS and NICOMEDES, ſubſequent geometers, for the quadrature of the circle, from which it obtained its name. But he obſerves that another geometer, SPORUS, was not ſatisfied with this application of that curve, and he gives ſome detail of the objections propoſed by SPORUS; and particularly, that in the conſtruction of this line the problem to be reſolved by it is in ſome meaſure aſſumed. PAPPUS however, beſides the common deſcription of its origin and of its properties, gives what he conſiders as a more ſtrictly geometrical deſcription, by *Loci ad Superficiem.**

After ſeveral Propoſitions on this ſubject, there is a repetition of the ancient claſſification of lines, and nearly in the ſame words† in which it was ſtated in the third book. It ſeems to be introduced here for illuſtrating the problem of the triſection of an arch of a circle, as in the former book it had a more particular reference to the Deliacal problem.

* This is Prop. 28, which is ſo much corrupted in COMMANDINE as to be ſcarcely intelligible; and in Note G. at the end of the preceding Memoir is placed Dr. SIMSON's corrected edition of it. Another deſcription of it is given in Prop. 29.

† The Greek of this paſſage is added in Appendix II.

This problem is resolved by PAPPUS in various ways; first, by a solid inclination, to which Dr. SIMSON adds an improvement; then by an hyperbola without the inclination;* and he mentions that in another treatise† he had resolved it by means of the conchoid, of which the method is obvious.

He subjoins some linear problems, such as dividing an angle or arch of a circle in a given ratio; the construction of an isosceles triangle, of which each of the angles at the base shall have a given ratio to the angle at the vertex; which are easily resolved, by assuming the quadratrix or spiral, as proper means of geometrical solutions.

Besides the very important emendation of Prop. 28th and 29th of this book by Dr. SIMSON, there are various other necessary corrections in other Propositions, and some useful readings adopted by him from MS. BULL. which it is unnecessary to specify. At the end of this book, there is in MS. BULL. a very imperfect sketch of a problem said to have been used by ARCHIMEDES, in his attempt to find a straight line equal to the periphery of a circle; and also that this use of it had been animadverted on. It is not in COMMANDINE's translation, and in its present state is altogether unintelligible.§

* This Inclination is alluded to in Note F. p. 99.

† From the very brief notice of this work, it seems to have been a comment on the Analemma of DIODORUS: "καὶ ἡμεῖς ἐν τῷ εἰς τὸ ἀνάλημμα Διοδώρου τρίχα τεμεῖν τὴν "γωνίαν βουλόμενοι κεχρήμεθα τῇ προειρημένῃ γράμμῃ." MS. BULL. Vide COMMAND. PAPP. fol. 56, a. l. 6.

§ This Proposition, in the same unintelligible state, is found in Savil. MS. No. 3, and at the end of the fourth book.

§. IV. *The Fifth Book.*

To the fifth Book there is a preface, in which PAPPUS makes ſome philoſophical obſervations on the curious inſtincts of animals, which in many caſes ſupply the place of reaſon. He mentions more particularly the inſtinct of bees, by which they conſtruct the receptacle for their proviſion on geometrical principles, by employing the hexagon, which of the three equilateral and equiangular figures that occupy the ſpace round a point, ſupplies, with the ſmalleſt labour, the moſt convenient accommodation. This obſervation ſeems to be introductory to the purpoſe of the book, which, at the end of the preface, he ſtates to be; to prove that of plane, figures which are equilateral and equiangular, and have equal perimeters, the greater ſpace is contained by the figure with the greater number of ſides; and that of all plane figures, of equal perimeters, the circle is the greateſt.*

This ſubject of iſoperimetrical figures is largely treated of in this book, containing fifty-ſeven Propoſitions.‡ Beſides

* The title of the fifth book in MS. BULL. is the following, Πάππου Ἀλεξανδρέως συναγωγῆς ε̄, περιέχει δὲ συγκρίσεις τῶν ἴσην περίμετρον ἐχόντων ἐπιπέδων σχημάτων πρὸς ἄλληλά τε καὶ τὸν κύκλον,|| καὶ συγκρίσεις τῶν ἴσην ἐπιφάνειαν ἐχόντων στερεῶν σχημάτων πρὸς ἄλληλα καὶ τὴν σφαῖραν.

|| Note in MS. BULL. it is erroneously τῶν κυκλῶν, but in the Savil. MS. No. 3. it is τὸν κύκλον.

‡ In Prop. 1. an eaſy Propoſition is aſſumed, which is demonſtrated in a Lemma (Note B. on that Prop.) by COMMANDINE. It is remarkable, that a Propoſition of the ſame import in Greek is found on the margin of MS. BULL. And in the text of MS. SAVIL. No. 3, is a ſimilar Propoſition. It would require however the examination of other MSS. to aſcertain the hiſtory of this Lemma.

the object mentioned in the preface, several other things are considered. It is proved, that of plane figures with equal perimeters, the greatest is that which is equilateral and equiangular. This principle is extended to solids; the regular solids (as they are called) are compared, and it is proved, that, of those with equal surfaces, the greatest is that with the greater number of sides. These Propositions are meant no doubt to be introductory to a demonstration of the Proposition, that of all solids with equal surfaces the greatest is the sphere. There are many references to ARCHIMEDES, and several of the Propositions in his treatise of the Sphere and Cylinder are demonstrated in this book by PAPPUS,* and, as he mentions, in a different manner.

It was formerly observed, that a number of Propositions in it, respecting the comparison of the five regular bodies having equal surfaces, were demonstrated only synthetically from the brevity and the facility of communication in that method; and that this explains the omission of the analysis in many ancient Propositions, though there can be no doubt of that analysis having been used by the authors in the investigation of them. He introduces the Propositions respecting these regular solids by a general dissertation on them before Prop. 18. in which he professes great reverence for the doctrines of the divine PLATO, in whose school it is supposed that these bodies were first defined, and their relations fully treated of. He

* In Prop. 35. (fol. 96. COMMAND.) is a very material improvement of the demonstration by Dr. SIMSON, by which the long note (Z) of COMMANDINE becomes unnecessary. This may be remarked in many of the Doctor's corrections, which, even when short, often supersede the use of long and sometimes tedious comments by COMMANDINE.

remarks, that it is aſſumed by philoſophers, without proof, that the ſphere is the greateſt of ſolids, having equal ſuperficies: he proceeds to give ſome popular illuſtrations of that truth; and in Prop. 18. he propoſes a compariſon of the ſphere with the regular ſolids. In the end of the book he ſtates, what is eaſily aſcertained, that there can be only five regular ſolids, which are comprehended by equal, ſimilar, and equilateral polygons, viz. the tetraedron, cube, octaedron, dodecaedron, and icoſaedron; in ſome of the Propoſitions Dr. SIMSON has remarked ſeveral proper and neceſſary corrections.

§. V. *Sixth Book.*

THE ſixth Book of PAPPUS, as the title intimates,‡ is employed chiefly in explaining and correcting ſome Propoſitions of THEODOSIUS, and ſome other ancient writers, in treatiſes containing chiefly, what is popularly called, the doctrine of the ſphere. In a ſhort preface the object of this book is ſtated, with a reference to three examples of Propoſitions criticiſed in it, viz. the 6th, iii. THEODOSIUS *on the Sphere*; the ſecond Prop. of EUCLID's *Phænomena*; and the fourth Prop. of THEODOSIUS *on Days and Nights*. This collection of treatiſes, obtained, in the Alexandrian ſchool, the name of *μικρὸς ἀςρονόμος*, or, as it is in PAPPUS *μικρὸς ἀςρονομούμενος*. Among the Arabians they were called *Libri intermedii inter* ςοιχειωτὴν *et magnum conſtructorem*, that is, between EUCLID and PTOLEMY:

§ The title of this book in MS. BULL. is Πάππου Ἀλεξανδρέως συναγωγῆς ἕκτον, περιέχει δὲ τῶν ἐν τῷ μικρῷ ἀστρονομουμένῳ θεωρημάτων ἀπορῶν λύσεις.

and in contraſt to the greater work of the latter (the Almageſt, ſometimes called μέγας ἀστρονόμος) theſe ſmaller tracts got the name of μικρὸς ἀστρονόμος.

According to VOSSIUS, this collection contained the nine following works, viz. THEODOSII *Sphærica*, EUCLIDIS *Optica*, ejuſdem *Phænomena*, THEODOSII *de Habitationibus*, ejuſdem *de Noctibus et Diebus*, AUTOLYCUS *de Sphæra*, ejuſdem *de Ortu et Occaſu*, ARISTARCHI *de magnitudinibus et distantiis Solis et Lunæ*, HYPSICLIS ἀναφορικὸν, ſive *de ascensionibus*.† FABRICIUS, when ſtating this collection, includes in it alſo MENELAI *Sphærica*, and EUCLIDIS *Data* et *Catoptrica* :‡ and he adds, that theſe treatiſes, or moſt of them, are often found together in Greek MS. in the libraries of Italy and France.

It may be alſo obſerved, that all theſe treatiſes, except that of HYPSICLES, are quoted or alluded to in this ſixth book, in which PAPPUS examines ſeveral ἐνστάσεις *(Instantiæ)*, to be found in them. By this term he appears to underſtand the objections to certain Propoſitions in theſe writers, or the limitations and exceptions neceſſary to be made reſpecting them, but which had been omitted by the reſpective authors. PROCLUS gives an explanation of this term;§ and from his diffuſe and not very ſatisfactory account, this word ἔνστασις *(Instantia)* ſeems to have been applied to any objection made to a Propoſition, or even to the denial of it, as preparatory to an indirect demonſtration of it, by proving the abſurdity of

† GER. VOSSIUS *de Scientiis Mathematicis, et Chronologia Mathematicorum*, cap. xxxiii. §. 18.

‡ FABRICII *Biblioth. Græc.* Harles, Hamb. 1795. tom. iv. p. 16, and alſo p. 212. SAVILIUS in Prælect. II. in EUCLIDEM.

§ PROCLUS, p. 58. HERVAGII; p. 121. BAROCII.

that denial. In PAPPUS, however, the application ſeems to be confined to the former meaning, with which the examples which he gives, from THEODOSIUS and others, correſpond.†

Prefixed to Prop. 39 of this book, is a long diſcuſſion concerning the poſitions in the before mentioned tract of ARISTARCHUS SAMIUS and this, with the 39th, 40th, and 41ſt Propoſitions on the ſame ſubject, is inſerted by Dr. WALLIS in his edition of *Ariſtarchus* publiſhed at Oxford in 1688. He adds the Greek of this extract from PAPPUS according to the two Savilian manuſcripts, which he takes an opportunity of characterizing.*

In the explanation and correction of ſeveral paſſages in theſe tracts by PAPPUS are introduced ſome curious Propoſitions, by which the doctrine of the ſphere of that period was improved; and there are ſome Propoſitions relating to a ſort of perſpective or projection of the ſphere, which are connected with Propoſitions of EUCLID's *Phænomena*. It may be alſo obſerved, that in four Propoſitions, viz. 31, 32, 33, 34, which are preliminary to ſome following Propoſitions reſpecting the ſphere, ſome diſtinctions of magnitudes are ſtated as examples, which may be either increaſed or diminiſhed without limit; or may be increaſed indefinitely,

† Examples of *inſtantiæ* may be found in Prop. 21, of this ſixth book, to which the preceding ten Propoſitions refer. Alſo in Prop. 51, (fol. 145. a. at the bottom, COMMAND.) Other criticiſms on EUCLID's *Phænomena*, and alſo on HIPPARCHUS, are mentioned in ſubſequent Propoſitions of this book. There are alſo many references to PTOLEMY.

* In his *præfatio ad Lectorem* he obſerves, "quorum (ſc. MSStorum) qui "elegantius ſcribitur (ſc. No. 8.) eſt mendoſior; quique feſtinantius et minus "eleganter ſcribitur (ſcil. No. 9.) eſt emendatior."

while there is a limit to the decreaſe, and converſely. And I mention them here, merely to remark the inaccuracy of COMMANDINE's tranſlation, by introducing the term *infinite* without authority from the original. Dr. SIMSON obſerves that he had not conſidered this book ſo particularly as he purpoſed afterwards to do, but which he appears not to have accompliſhed: at the ſame time he had made ſome neceſſary and uſeful corrections in ſeveral Propoſitions as they ſtand in COMMANDINE's tranſlation.*

§. VI. *Seventh Book.*

THIS Book, as the title imports, is wholly employed on that curious ſubject, the ancient analyſis.†

THE collection of treatiſes known by the name of *τόπος ἀναλυόμενος*, next to the *Elements* and *Data* of EUCLID, were important for facilitating the reſolution of geometrical problems. They were compoſed, by the elder ARISTÆUS, EUCLID, and APOLLONIUS;‡ but at what time the collection

* It may be obſerved, that in the MS. BULL. and in the Savil. MS. No. 3, are ſome marginal notices and at the end of Prop. 59, a ſcheme reſpecting climates which may deſerve conſideration; in that Propoſition alſo are references to PTOLEMY.

† The title of the 7th book, from one of the Pariſian MSS. No. 2368, is as follows; *Πάππου Ἀλεξανδρέως συναγωγῆς ζ ὃ περιέχει τὴν τάξιν, καὶ τὴν περιοχὴν καὶ τα λήμματα τοῦ ἀναλυομένου τόπου.*

‡ In this firſt ſtatement of the authors in the preface to this viith book theſe three only are mentioned. In a ſubſequent more particular account (alſo in this preface) PAPPUS mentions two books on the *Medietates* by ERATOSTHENES. But neither this work of ERATOSTHENES, nor the *Loci* of ARISTÆUS, nor the *Loci ad Superficiem* by EUCLID, are deſcribed by PAPPUS.

obtained the name by which they are diſtinguiſhed in the preface to this book, is unknown. They muſt have been compoſed at different periods, from the known interval of the times of the reſpective authors, who all lived above 500 years before the age of PAPPUS. The expreſſion of PAPPUS is not definite; yet it ſeems to imply that the collection was made, and the name impoſed, before his own time, for his expreſſion ſeems to ſuppoſe the order of the books as exiſting.‡

The preface to this book, one of the moſt valuable remnants of ancient geometry, contains a particular deſcription of the nature and contents of a certain number of theſe treatiſes, which, both from the ſubjects of them, and from their being thus diſtinguiſhed by PAPPUS, may be conſidered as by far the moſt important part of the collection; and the book itſelf contains a number of Lemmata or elementary Propoſitions aſſumed (probably without proof) in the treatiſes deſcribed in the preface, but of which the demonſtrations are added by PAPPUS.

PAPPUS, however, premiſes a ſhort but accurate account of the ancient method of analyſis and ſyntheſis; of which a free tranſlation will be more ſatisfactory than any abridgement, for conveying a correct notion of this curious branch of ſcience, ſo much valued among the ancients, and, till the

‡ Dr. HALLEY, whoſe authority in ſuch points is very great, at the end of his preface to the *Sectio Rationis*, intimates an opinion that the collection was made by PAPPUS. But the circumſtances now mentioned render this inference from the words of PAPPUS at leaſt doubtful; and Dr. SIMSON has remarked another miſtake of Dr. HALLEY's in the before mentioned preface reſpecting the principal object in forming this collection.

time of Dr. Halley and Dr. Simson, fo little underftood among the moderns.

It is employed both for the refolution of problems, and alfo for inveftigating the truth of theorems either afferted or conjectured to be true; though it is in the former clafs of Propofitions that the ufe and importance of this method is chiefly known.

In the analyfis of a problem, what is propofed to be done, or to be found, is fuppofed to be obtained; and the confequences of fuch an affumption are fucceffively deduced by means of all previoufly known practical Propofitions, which appear to be connected with it, till at length we arrive at fome conclufion, which, from the ftate of geometrical fcience at the time, we know to be practicable; and thence the folution of the problem is deduced Synthefis, or compofition is the reverfe of analyfis; and by affuming the laft practicable confequence of the analyfis, we proceed in a contrary order through the feveral fteps of that analyfis till we neceffarily reach the conftruction of the thing required, and then the problem is refolved. In this analyfis however, if we arrive at a confequence which we know, from previous Propofitions, to be impoffible, then the problem propofed muft itfelf be impoffible; and further, if we find, from the progrefs of the compofition, that in certain relations of the given magnitudes the conftruction is practicable, while in others it becomes impoffible; the afcertainment of thefe relations becomes a neceffary part of the folution, and is called the determination of the problem.

In the analyſis of a theorem,* the aſſertion in the enunciation is aſſumed as true; and by reaſoning from it, by the application of known geometrical Propoſitions which appear to be connected with it, we trace the ſucceſſive conſequences of the firſt aſſumption, till we arrive at ſome one which we know to be true, or to be falſe. If we arrive in this manner at a true Propoſition, by proceeding from it, in a contrary order, through the ſeveral ſteps of the analyſis, we ſhall neceſſarily arrive at the propoſed aſſertion; and this laſt proceeding will be a demonſtration or compoſition of the theorem. In like manner, if in this inveſtigation we arrive at a concluſion which we know to be falſe, from the ſame neceſſary concatenation of Propoſitions, we infer the falſehood of the aſſumed aſſertion; and if it were requiſite, we might alſo demonſtrate that inference. But it not being my deſign, in this general account of the ſeventh book of PAPPUS, to attempt a full expoſition of the ancient analyſis, which, for the uſe of thoſe who have not conſidered the ſubject, would require alſo the illuſtration of examples, I can only refer to thoſe treatiſes where ſuch explanatory examples of the practice of analyſis may be found.† I ſhall

* The analyſis of theorems is often uſeful. A theorem may be propoſed to a geometer, that he may inveſtigate the demonſtration; it may be only ſuſpected to be true from analogy, from the apparent relations of magnitudes in geometrical diagrams, and even from accident; but in all ſuch caſes an analyſis will aſcertain the truth or falſehood of the aſſumed Propoſition.

† In the remaining works of APOLLONIUS, as publiſhed by Dr. HALLEY, and alſo in PAPPUS, many problems and theorems are treated analytically. In Dr. SIMSON's different works are various examples in the pure ſtile of the ancient geometry. The analyſis of theorems is well illuſtrated in Dr. STEWART's *Propositiones Geometricæ more veterum demonstratæ*. For problems, I may refer to Dr. HORSLEY's (Biſhop of St. Aſaph) *Delectus Problematum*; and to Mr. Profeſſor LESLIE's *Geometrical Analysis* in his *Elements of Geometry*. In that treatiſe the general problems of APOLLONIUS, mentioned in this book of PAPPUS, are introduced as examples.

therefore only further obferve, that, in this practice, fome of the principles and rules commonly laid down by writers on the application of the algebraical analyfis to the folution of geometrical problems, may here alfo be ufefully employed; though feveral of them, no doubt, are more peculiarly fitted for the modern fyftem.†

After this general account of the ancient analyfis, PAPPUS, in his Preface, proceeds to enumerate the books contained in the τόπος ἀναλυόμενος, and afterwards to give a particular account of the nature and contents of a certain number of them, viz. of twenty-four; the whole number being thirty-three.‡

Thefe twenty-four books are, I. *Data* of EUCLID; III books *De Porifmatis*, alfo by EUCLID; and the following twenty by APOLLONIUS: II. *De Sectione Rationis*; II. *De Sectione Spatii*; II. *De Sectione Determinata*; II. *De Tactionibus*; II. *De Inclinationibus*; II. *De Locis Planis*; and VIII. *De Conicis*.

Thefe books, though all ufeful in facilitating the folution of geometrical problems, yet are of different characters, and promote the object of the whole collection in different ways; as may appear from the many references to them in this Memoir, of which the following fhort recapitulation is added.

† See NEWT. *Arith. Univerfalis*, cap. 23.——See alfo fome obfervations of Dr. SIMSON on the ancient analyfis, in Note K. at the end of the Memoir, p. 121—128.

‡ In this preface PAPPUS mentions his purpofe of giving a defcription of thefe books as far as the *Conics* of APOLLONIUS; and therefore, according to his arrangement, which is not in the order of time, he appears to omit by defign the nine following books: V. of ARISTÆUS *de Locis Solidis*, (which were no doubt fuperfeded by the *Conics* of APOLLONIUS;) II. of ERATOSTHENES *de Medietatibus*; and II. by EUCLID, *de Locis ad Superficiem*.

I. *The Data of* Euclid. This book, one of the few which have been preferved in the original language, may be confidered as a collection of elementary problems; and the demonftrations as given by Euclid, would become the analyfes of thefe Propofitions, had they been enunciated as problems. The book was conftantly ufed by the ancients in their refolutions of problems, though it was not their practice to quote the particular Propofitions; and this application of the Data was continued among fuch of the moderns as follow ftrictly the ancient method of analyfis, and is now fo well known, that it is unneceffary to give any detail of it.* The *Sectio Rationis, Sectio Spatii, Sectio Determinata*, and the Treatifes *De Tactionibus*, and *De Inclinationibus*, are all general problems of frequent recurrence in geometrical inveftigations, and were refolved by Apollonius in the moft complete manner, all the poffible cafes being diftinguifhed; and of each cafe a feparate analyfis and compofition are given, with the refpective determinations in all thofe cafes which required them.†

The ufe of thefe general problems, as has been repeatedly mentioned, was for the more immediate refolution of any propofed geometrical problems which could be eafily reduced to a particular cafe of any one of them.‡ By fuch a

* See in Note K. at the end of the Memoir, fome remarks on the Data, in the correfpondence between Dr. Simson and Mr. Scott.

† Pappus, in this preface, alluding to a problem connected with the Treatife of Tactions, ftates with brevity, but with precifion, what is requifite in a perfect folution of fuch general problems: "*καὶ ταῦτα ἀναλῦσαι, καὶ συνθεῖναι, καὶ διορίζεσθαι κατὰ πτῶσιν.*"

‡ When a problem can be refolved with equal facility by the ufe of known elementary Propofitions, a reference in fuch a cafe to any of thefe general problems becomes unneceffary.

reduction the proposed problem was considered as fully resolved; because it was then necessary only to apply the analysis, composition, and determination of that case of the general problem to this particular Proposition, which was shewn to be comprehended in it. The apparatus of separate solutions, with the determinations of every possible case which is essential to the use of these general problems, may appear forbidding; and if regularly perused without examples of the application, may sometimes appear tedious and uninteresting; and this perhaps, may have created some prejudice against the study of them. Dr. HALLEY, indeed, seems to think that the books of the *τόπος ἀναλυομένος*, were intended merely for the instruction of beginners in the study of the geometrical analysis:§ but it is justly observed by Dr. SIMSON, that though they may be most advantageously employed for that purpose, yet that it is manifest from the contents of those books, of which he gives some detail, and also from the manner in which they are referred to in the small remains which we possess of the ancient geometry, the chief object of them was what has now been stated. Even in PAPPUS there are some examples of problems being resolved by a reduction of them to cases of these general problems. In the 85th Prop. of this seventh book, a problem, useful in the *Treatise of Inclinations*, is by analysis brought to a case of the *Sectio Determinata*, as was formerly mentioned in the Memoir. Another example is Prop. 164, of the same book, the last lemma belonging to the Porisms; which is a problem resolved by PAPPUS, by reducing it to a case of the *Sectio Spatii*, and the composition of the problem is considered as compleated, merely

§ HALLEII *Sect. Rationis*, Præfat. See also *Loci Plani*, SIMSON's Præfat. p. vii.

by a reference to the conſtruction of that caſe. This tract of APOLLONIUS being loſt, and no reſtoration of it having been made in the time of COMMANDINE, the caſe referred to is reſolved by him, though in a tedious manner, which may be compared with the ſolution of it in Dr. HALLEY's reſtoration of that work.*

The other books, deſcribed in this preface by PAPPUS, are uſeful for the ſame purpoſe of facilitating the ſolution of geometrical problems, though in a manner different from what has juſt been mentioned of the preceding treatiſes. The treatiſes of *Loci* in particular are very important, and in a method well underſtood by thoſe even ſlightly acquainted with either the ancient or modern geometry. The *Loci Plani* of APOLLONIUS are uſeful for the ſolution of plane problems; and often alſo, with *Loci* of a higher order, may be required in the ſolution of ſuperior problems. Though the original work of APOLLONIUS be loſt, the elegant and compleat reſtoration of it by Dr. SIMSON leaves nothing to be regretted on that ſubject. The *Loci Solidi* of ARISTÆUS might have been very ſerviceable in the ſolution of ſolid problems, but it has periſhed; and probably, even in the time of PAPPUS, had been ſuperſeded by the great work of APOLLONIUS on *Conic Sections*, from which not leſs important aid might have been derived for reſolving that claſs of problems.†

* SCHOOTEN alſo reſolves this problem, *Exercit. Mathem.* p. 104, 105, and ſeems to blame PAPPUS for not reſolving it directly, but by a reference to a caſe of the *Sectio Spatii*; from which it appears, that SCHOOTEN was not appriſed of the true uſe of theſe ancient books. His Propoſition alſo is not ſo general as it is ſtated in PAPPUS. See Dr. SIMSON's edition of this Prop. *Opera Reliqua*, p. 529.

† As ſeven books of the *Conics* of APOLLONIUS have been recovered, and the eighth reſtored in a ſatisfactory manner by Dr. HALLEY, it is unneceſſary to enter

The porifms of EUCLID were a peculiar clafs of Propofitions, and ufed among the ancients in the folution of fome of the moft difficult geometrical problems. The long account of them in this preface has been particularly unfortunate in fuffering from the injuries of time; fo that till Dr. SIMSON's perfevering induftry and ingenuity were employed on them, no fatisfactory explanation of the nature of thefe Propofitions could be difcovered, nor could any fingle Propofition of EUCLID's treatife be reftored. But in the preceding Memoir fo full an account of Dr. SIMSON's reftoration of them has been given, that it is unneceffary in this place to make any further obfervations refpecting them.

With refpect to the *τόπος ἀναλυομένος*, I fhall only further obferve, that on the revival of mathematical learning in Europe, if the ftudy and application of the ancient analyfis had continued, without being nearly fuperfeded by the ufe of the modern algebra; it is highly probable that later geometricians would, from time to time, have made additions to the ancient collection; and would have inveftigated various other general problems, with compleat folutions of their feveral cafes, for the fame general purpofe for which thefe ancient books were compofed by EUCLID and APOLLONIUS.

At the end of the defcription of the *Conics* of APOLLONIUS, in this preface to the feventh book of PAPPUS, there are fome interefting obfervations of a more general nature, which have

into any difcuffion refpecting the account of this work by PAPPUS. It is, however, both curious and interefting; and Dr. HALLEY, in his preface to the corrected edition of this preface to the feventh book of PAPPUS, makes fome valuable obfervations on it. Dr. SIMSON adds fome ufeful notes on the *Lemmata* of this work.

been particularly remarked by Dr. HALLEY. I allude to the account of the celebrated ancient problem of the "*Locus ad* "*tres et quatuor rectas*;" and the extenſion of it mentioned by PAPPUS to the caſes where there are a greater number of ſtraight lines than four, to which lines are drawn in given angles from the point, of which the *Locus* is to be inveſtigated. The original *Locus* is ſolid, but theſe other caſes produce *linear Loci*, that is, curves of ſuperior orders; which, however, had not been inveſtigated in the time of PAPPUS. In ſome general obſervations on the ſubject by PAPPUS, it may be remarked that he mentions the method of expreſſing dimenſions above the cubic, by means of compound ratios.

This laſt article of the preface is concluded by a diſtinct enunciation of the celebrated theorem of GULDINUS, which, twelve centuries after the age of PAPPUS, excited much curioſity and admiration. Whether this theorem was invented by PAPPUS, or by ſome other geometer, is not ſtated. It is proper to remark, however, that no imputation is conveyed on the originality of the diſcovery by GULDINUS; as in COMMANDINE's tranſlation, which was the only account then in print, the Propoſition is altogether unintelligible.* But as this portion of the preface has not been conſidered in Dr. SIMSON's notes,

* The improved tranſlation of this preface of PAPPUS, by Dr. HALLEY, appeared only in 1706, long after the time of GULDINUS; and in it this theorem from PAPPUS was firſt intelligibly publiſhed. MONTUCLA indeed remarks, (tom. i. p. 325,) that the work of GULDINUS was publiſhed before the ſecond edition of COMMANDINE's PAPPUS in 1660. But that edition could not have given him any aid, for it contained no valuable improvement of the former; and in this particular point, it is an exact tranſcript of the paſſage in the firſt edition.

it becomes unneceſſary in this place to enter into any further diſcuſſion of it.

Having given ſo particular an account of the preface of the ſeventh book, (certainly one of the moſt curious remains of ancient geometry,) a very ſhort ſtatement of the contents of the book itſelf will be ſufficient. It conſiſts of a great number of Propoſitions, (238,) ſome of which are problems, but the greater part theorems. They are called *Lemmata*, and divided into claſſes according to the ſeveral treatiſes deſcribed in the preface, to which they reſpectively belong. They appear in general to be elementary Propoſitions aſſumed, but we may ſuppoſe not demonſtrated, in theſe ancient books, from which they were collected probably by PAPPUS; and the collector has added the demonſtrations. Some of them indeed are curious and important Propoſitions; but the greater number of them are chiefly valuable for the aid which they give in the attempts of modern geometers in reſtoring accurately the loſt books of the τόπος ἀναλυόμενος. In ſuch attempts, it was an intereſting object, not merely to reſolve the general problems contained in that collection, with their caſes and determinations, but alſo, where it was poſſible, to follow the particular mode of ſolution employed by the original authors. Theſe Lemmata‡ preſerved by PAPPUS have

‡ According to PROCLUS, in his comment on Prop. 1. I. Elem. a Lemma (ſumptio BAROC.) is an aſſumption of a geometrical truth, in the demonſtration of a Propoſition; which truth, however, has not been demonſtrated by known geometrical writers. In this ſenſe is the term uſed by PAPPUS, and the Lemmata of the ſeventh book ſeem to be of that deſcription. It appears from the books of APOLLONIUS's *Conics* ſtill preſerved, that the Lemmata of PAPPUS belonging to

ſupplied the means of doing this; and when proper analyſes and compoſitions were diſcovered, which required alſo the uſe of the particular Lemmata mentioned by PAPPUS, there was a high degree of probability, that ſuch ſolutions were truly thoſe given by EUCLID and APOLLONIUS.

Many neceſſary corrections of theſe Lemmata, as they appeared in COMMANDINE's tranſlation, were made by Dr. SIMSON. Some of the corrections were ſupplied by the Pariſian MS. of PAPPUS, of which Dr. MOOR procured a copy, as has formerly been mentioned; and they are remarked in the Doctor's reſtorations of the *Loci Plani*, and *Sectio Determinata*. In the Lemmata belonging to the Poriſms, which are all given in his poſthumous work on that ſubject, and in an improved ſtate, are many important emendations; but it does not appear

that work were aſſumed by him, in the manner now mentioned; and Dr. HALLEY, in his edition of APOLLONIUS, gives theſe Lemmata from the Savilian MSS. of PAPPUS. In the demonſtrations of the Propoſitions of APOLLONIUS, Dr. HALLEY refers occaſionally to thoſe Lemmata, as the want of them appeared in the text for rendering the demonſtrations compleat. It appears likewiſe from the ſame edition, (p. 97 of the tranſlation of vth, vith, and viith, books) that ABDOLMELEC SCHIRAZITA, an Arabian epitomizer of the *Conics*, had collected ſome other Lemmata, (8.) beſides thoſe in PAPPUS, which were aſſumed without proof in the demonſtrations of the ſeventh book, and which therefore he prefixes to that ſeventh book; and they are placed by Dr. HALLEY immediately after the Lemmata of PAPPUS belonging to it.

In modern times, however, a Lemma is underſtood to be an eaſy preliminary Propoſition, which might have been incorporated with the more important one to which it is prefixed, but is more conveniently detached from it, and ſometimes is itſelf a Propoſition deſerving notice. In the treatiſe of ARCHIMEDES on the Sphere and Cylinder (before Prop. 17. b i.) is an eaſy Lemma demonſtrated by himſelf; and at the end of Prop. 17th are ſeveral Lemmata, which he ſtates to have been demonſtrated by thoſe who preceded him; and are Propoſitions of the xiith b. of EUCLID. They are in ſubſequent Propoſitions aſſumed as known, but a formal quotation of preceding works was not the practice of the ancient geometers.

that any of thefe laft were derived from that MS. as he never quotes it; and moft of them, indeed, were made before Dr. MOOR obtained that copy of the feventh book.‡ A confiderable number of the Lemmata of the feventh book (about 90) have been publifhed, as corrected by Dr. SIMSON; and the attention and accuracy with which he has examined thefe Propofitions, will be obvious to the intelligent reader, by comparing them with the edition of them in COMMANDINE's tranflation. And though the remaining Lemmata in this book do not appear to have been fo particularly confidered by the Doctor, yet there are in his notes on them many neceffary corrections, with ufeful explanations, fome of which have been mentioned in the notes added to this memoir.

The Lemmata in this feventh book belong to the feveral treatifes defcribed in the preface; according to the annexed arrangement from COMMANDINE's tranflation; in which enumeration, however, are fome irregularities, as is obferved by Dr. SIMSON.

‡ In Dr. SIMSON's PAPPUS is a memorandum for enquiry if the copy of the *Codex Regius* contained any of the neceffary emendations which he had made on a Lemma (Prop. 130.) of the Porifms, but no mention is made of the refult; which is a further prefumption that the Doctor, though he had had the ufe of the MS. for correcting the Lemmata of the two treatifes above mentioned, yet that he had not had an opportunity of confulting it refpecting the Porifms.

Treatises.				No. of Lemmata.
No. 1.	APOLLONIUS,	*De Sectione Rationis et Spatii,* from beginning to Prop. 21 incl.		21
No. 2.	———	*De Sectione Determinata,*	to Prop. 64.	43
No. 3.	———	*De Inclinationibus,*	to — 95.	31
No. 4.	———	*De Tactionibus,*	to — 118.	23
No. 5.	———	*De Locis Planis,*	to — 126.	8
No. 6.	EUCLIDES,	*De Porismatis,*	to — 164.	38
No. 7.	APOLLONIUS,	*De Conicis,*	to — 234.	70
No. 8.	EUCLIDES,	*De Locis ad Superficiem,*	to — 238.	4*
				238

Of these Lemmata, since COMMANDINE's translation, have been published:

By Dr. HALLEY, Greek and Latin,	No. 1, and 7.	- -	91
By Dr. SIMSON, in Latin, and corrected,	Nos. 2, 5, and 6.		89
By CAMERER, Greek and Latin,	No. 4.	- - - -	23
			203

A few others have been mentioned in this Memoir as corrected by Dr. SIMSON and others.

It is proper also to mention in this place, that a Proposition of some curiosity, though not in COMMANDINE's translation, is found at the end of this book in the two Parisian MSS. and also in those belonging to the Savilian Library at Oxford. It is entitled λῆμμα τȣ̃ ἀναλυομένȣ τόπȣ; and an accurate copy of it, from the Parisian MSS. is placed in Dr. SIMSON's PAPPUS, from which a translation by the Doctor is added.

* Four only in COMMANDINE's translation are numbered as Propositions, though it seems that five Lemmata were intended. And it is also to be remarked, that the four last lines of fol. 300. a. COMMAND. though printed in the character of the Commentary, are part of the text, of which the Greek is to be found in Savil. MS. No. 3. Dr. SIMSON, however, does not appear to have considered these Lemmata; at least he has left no notes respecting them. See the Memoir, p. 48.

" Propositio PAPPI ALEXANDRINI quam ex COD. MSS.†
" Bibliothecæ Parisiensis paucis abhinc Septimanis exscripsit
" D^{nus} JACOBUS MOOR, collega meus, 7mo Nov^{ris} 1748.

" Si latera circa angulum rectum trianguli BAC, (Fig. 10.)
" viz. BA AC, secentur in K, L, ita ut tam BK ad KA quam AL
" ad LC sit ut BA ad AC, et jungantur BL CK, sibi mutuo
" occurrentes in F; et ad basim BC ducatur AFG, erit AG ad
" BC perpendicularis.

" Ducatur enim CE parallela ipsi BA occurratque ipsis BF,
" AG, (productis) in E, H; et puta verum esse theorema, sc.
" angulum AGB rectum esse: æquiangula igitur sunt BAC
" ACH triangula, quia ABC angulus æqualis est ipsi GAC seu
" HAC (8 6.) et recti sunt anguli BAC, ACH; ut igitur BA ad
" AC, ita AC ad CH. Est autem propter parallelas EC ad CH
" ut (BK ad KA hoc est ex hypothesi, ut BA ad AC hoc est ut)
" AC ad CH: æquales igitur sunt EC, CA. Et propter parallelas
" est AL ad LC ut BA ad CE, hoc est ad AC, quod quidem
" verum est ex hypothesi.

" Componetur vero ita. Eadem manente constructione,
" quoniam ex hypothesi est BA ad AC ut AL ad LC, hoc est
" ut BA ad CE; erunt AC, CE, æquales. Est itidem ex hypo-
" thesi BA ad AC ut (BK ad KA hoc est ut) CE seu AC ad
" CH: ergo (6. 6.) æquiangula sunt triangula BAC, ACH, et
" angulus ABC æqualis ipsi HAC seu GAC, et in triangulis ABC,
" GAC communis est angulus ACB, ergo reliquus AGC æqualis
" est reliquo BAC, sc. angulo recto. Q. E. D.

" N. B. In Prop. PAPPI facile ostenditur AK, AL, equales esse."

† " Viz. No. 2368, et No. 2440.

In the ſame MS. volume from which the preceding Propoſition is copied, there is the following different ſtatement of it:

[The ſame figure.]

"Si ſit triangulum ABC rectum habens angulum BAC, "ducatur autem BD parallela ipſi AC æqualis autem rectæ "BA; et CE parallela ipſi BA æqualis autem ipſi AC; et CD "BE jungantur ſibi mutuo occurrentes in F, et juncta AF "occurrat baſi BC in G, erit AG perpendicularis ad BC."

This Propoſition, which is equivalent to the preceding, was ſuggeſted by the figure of the 47 Prop. I. EUCL. and had been demonſtrated by Dr. SIMSON, before he met with the other. There is no intimation in the manuſcripts hitherto examined of the purpoſe for which this Propoſition was placed at the end of this book; and though it is called a lemma of the τόπος ἀναλυομένος, there is no apparent connection between it and any of the treatiſes deſcribed in this book, or with any of the *Lemmata* belonging to them; but from its being found in ſo many manuſcripts of PAPPUS, there is a preſumption of its having been placed in the *Mathematical Collections* by him.

§. VII. *The Eighth Book.*

THE object of this laſt book* is to give ſome account of the ancient mechanics; and though a curious document of the ſtate of that branch of ſcience in the time of PAPPUS, yet from

* The title from MS. BULL. is "Πάππου Ἀλεξανδρέως συναγωγῆς ηον περιέχει δὲ "μηχανικὰ προβλήματα σύμμικτα§ ἀνθηρά."

§ σύμμικτα is wanting in MS. Savil. No. 3.

the great improvement both in the theory and practice of mechanics in modern times, it is comparatively of little value.

The original genius of ARCHIMEDES was diſtinguiſhed in this department, as appears in part even from the portions of his numerous writings which have been preſerved, and alſo from the many references to his mechanical inventions in this book of PAPPUS, and in many other ancient writers. A conſiderable part of this book is employed in deſcribing what were then, and ſtill are, called the Mechanic Powers, and the moſt obvious combinations of them for the common purpoſes of life; eſpecially for raiſing up and for drawing very great weights.* PAPPUS avows its being chiefly borrowed from HERO, a diſtinguiſhed geometer and mechanician, whoſe works he frequently quotes, and ſome of which ſtill remain.† There is a long preface, which from ſome ſtatements of the mechanical notions, and of the arts of that period, becomes curious. It contains alſo ſome obſervations on the utility of mechanics, and of the connection of mechanics with geometry; in it alſo are diſtinguiſhed ſeveral branches of that ſcience, with notices of treatiſes on it which are loſt. Beſides many references to HERO, there are very ample teſtimonies in this preface of the fame of ARCHIMEDES, from his mechanical writings

* Towards the end of the book are deſcriptions of the ancient machines for theſe purpoſes; and in the MS. copies are deſigns of ſuch machines made from theſe deſcriptions. It may be obſerved, however, that thoſe drawings are different from each other, and from the engravings in COMMANDINE, which no doubt had been copied from ſketches in the MS. which he followed.

† This is the elder HERO, of Alexandria, who is ſuppoſed to have lived about fifty years after ARCHIMEDES.——GER. VOSSIUS.

and inventions,‡ in many of which his geometrical and arithmetical ſcience was employed. But as has already been remarked, the chief value of this book is from the information which it affords of the ſtate of mechanical ſcience in the age of Pappus. I muſt obſerve alſo, that though Dr. Simson makes a few corrections on this book, he ſeems never to have conſidered it particularly, and therefore it is unneceſſary to give a more particular account of its contents.§

§. VIII. *Concluſion.*

From the ſhort account which has been given of the ſeveral books of Pappus, the miſcellaneous nature of the Collection is ſufficiently apparent. Several ancient mathematicians are mentioned by him, of whom no other notice remains; and

‡ In Prop. 10th, he mentions the well-known obſervation of Archimedes, that with a fixed ſtation he could move the earth. And the reſolution of the general mechanical problem, "to move a given weight with a given power," he calls the 40th invention of Archimedes.

§ It is proper, however, to mention an obſervation of Dr. Simson on Prop. 14 of this book: "Addatur hic (poſt verbum *quidem* in linea quarta hujus paginæ, viz. fol. 320. b.) "*Geometrice* ut dudum emendavit Gregorius a St°. Vicentio, pag. 291." *Quad. Circuli.*——In the ſame place is alſo the following note, referring to the *Codex* Bull.; he ſays, "In quo poſt verba ὀργανικῶς εὑρεῖν, legitur μεθοδεύεται δὲ τὸν τρόπον τοῦτον. "Verbo vero μεθοδεύεται ſignificatur conſtructio geometrica." Gregory alſo makes another remark on Commandine's Note B. in the ſame page of Pappus, (1588.) The laſt ſentence of the enunciation of Prop. 12, (fol. 317. a) in Commandine's tranſlation is "Invenitur autem methodo inveſtigata hoc pacto;" but it ought to have been "inveſtigabitur autem geometrice ita;" the Greek (MS. Bull.) being "εὑρήσεται μεθοδευθὲν οὕτως." Dr. Simson alſo remarks, reſpecting the laſt ſentence of Commandine's Note B. on Prop. 14, viz. 'Mirum eſt Pappum,' &c. "Non mirum "quoniam ex elementis conicis ſatis hujus demonſtratio patet."

many curious particulars are detailed refpecting the ancient ftate of mathematics, and of the problems and theorems which engaged the attention of the geometers of thofe times. It is written with only a general attention to method; there is little uniformity in the ftile, and the work has probably been compofed at different times. Some propofitions are demonftrated minutely, and rather diffufely, while in others many important fteps are omitted,* as is often remarked by COMMANDINE, and thefe omiffions he has generally fupplied. As a further proof of this character of the *Collections*, the very frequent repetitions in different parts of it, of the fame Propofitions, fometimes with the fame, and fometimes with different demonftrations, may be mentioned.§ It is, however, a moft interefting work, and more efpecially from the account of the analytical geometry of the ancients in the feventh book; an edition of the Greek, from a collation of the many manufcripts which are known ftill to exift in the libraries of Europe, would be moft acceptable to all the admirers of the elegance and accuracy of ancient geometry; and for fuch an undertaking Dr. SIMSON's notes

* Dr. SIMSON makes a remark to this purpofe, *Opera Reliqua*, p. 526, at the end of Prop. 76. Many of the imperfections of PAPPUS, as they appear in the exifting MSS., muft be attributed to the tranfcribers.

§ The following examples may be remarked. The two ftatements of the ancient claffification of problems, in b. iii. fol. 4. b. and b. iv. fol. 60. The two accounts of the Conchoid in thefe two books. The two folutions of the Deliacal Problem, b. iii. fol. 7, and b. viii. Prop. 2, nearly in the fame words. In Dr. SIMSON's edition of the *Sectio Determinata*, feveral duplicates are mentioned. Other repetitions alfo might be pointed out, fuch as Props. 31, 57, 178, 194, book vii. Alfo Props. 87, 153, 206, in the fame book. Alfo Props. 23, 58, 193, b. vii. Several of thefe are remarked by Dr. SIMSON, in his notes, both printed and unpublifhed. See *Opera Reliqua*, pp. 409, 410.

would be highly valuable.‡ He does not feem to have taken a regular and accurate furvey of the whole, with any view to the immediate completion of fuch a work; nor has he confidered every paffage which would require correction or explanation; yet his emendations and illuftrations, feveral of which have been mentioned in this Memoir, will be an important repofitory of materials for the affiftance of any future editor.

It is proper likewife to obferve, that among Dr. SIMSON's notes are feveral generalizations of Propofitions in PAPPUS, and alfo fome connected Propofitions, which, though valuable in themfelves, may probably not be all confidered as properly belonging to a new edition of that author. Several alfo of the Doctor's notes contain corrections of COMMANDINE's commentary, moft of which might be omitted in conducting a new edition; of which the proper object muft be, to give the text of PAPPUS free from errors, and to introduce fuch comments only as are neceffary for explaining real difficulties, and for filling up material deficiencies in the demonftrations.

It may be alfo obferved, that from what is known refpecting feveral of the remaining MSS. and the fimilarity of their defects, it is much to be apprehended that few things of material importance, beyond what Dr. SIMSON has already remarked, are likely to be difcovered; though an accurate examination of thefe MSS. would certainly be moft defirable.

‡ By comparing Dr. SIMSON's MS. notes on the Lemmata of the feventh book with his corrected edition of thefe Lemmata in his reftorations of the *Loci Plani Sectio Determinata*, and *Porisms*, his tafte and judgment may be remarked in felecting only thofe notes which were neceffary, and omitting the others which he had written on his firft review of thefe Propofitions.

After every aid is obtained from remaining MSS. the ability and intelligence of an editor muſt be depended on for the beſt uſe of exiſting materials; and it is to be conſidered, that, in geometry, the nature of the ſubject, and the train of argument, may often enable an editor to ſupply the defects and correct the errors of manuſcripts, to which, indeed, thoſe on mathematical ſubjects appear to be more liable than any others.

The copy of COMMANDINE'S PAPPUS which Dr. SIMSON uſed, and in which he wrote his notes, was the firſt edition PISAURI, 1588, and VENETIIS, 1589, and to this all quotations in this memoir refer.

It is neceſſary alſo to mention, that a few years after the Doctor's death, an application was made to his executor Mr. CLOW, on the part of the Delegates of the Clarendon Preſs, in the Univerſity of Oxford, for a tranſcript of all the MS. notes in this volume, as they were left by the Doctor. This was readily complied with, on the reaſonable condition, that whatever notes and corrections of Dr SIMSON's might be adopted in a new edition of PAPPUS, they ſhould be particularly diſtinguiſhed and acknowledged as his. Some time after, Mr. CLOW depoſited this valuable *legacy* from his deceaſed friend, in the library of the College of Glaſgow; and having had, by the favour of the Principal and Profeſſors, every convenience for conſulting it, I have been enabled to give from it ſome intereſting particulars of Dr. SIMSON's geometrical ſtudies.

§. IX. *Of the MSS. of* PAPPUS.

A Confiderable number of manufcripts of the collections of PAPPUS are to be found in various libraries of Europe, but all of them which have been examined, are mutilated; and contain many errors, from the ignorance or carelefsnefs of the tranfcribers.†

COMMANDINE appears to have had the ufe of only one manufcript, of which the hiftory and fate are unknown; but from his numerous corrections of it in his comm ntary, in which he generally inferts the erroneous paffage of the MS. a judgement may be formed of its very deficient ftate. In it, as in feveral other MSS.‡ the two firft books are entirely wanting.

In feveral MSS. however, a portion of the fecond book (about one half) is preferved. This was found in one of the Savilian MSS. viz. No. 9, and publifhed by Dr. WALLIS, in 1688, with learned notes. It is underftood alfo, that one

† Dr. SIMSON, in a note on a Propofition, of the feventh book of PAPPUS, remarks, "non pauca autem in hac Propofitione, in eo codice (Parifienfe Regio fc.) " vitiata funt, ut in omnibus fere PAPPI Propofitionibus, et, ut videtur, in omnibus " manufcriptis."

‡ The fame is to be remarked of the Savil. MS. No. 3; alfo of a MS. of PAPPUS in the Neapolitian Royal Library, mentioned by FABRICIUS *Bibl. Græc.* tom. v. page 790, Hamburgh, 1798, &c. In the MS. BULLIALDI, afterwards to be mentioned, the two firft books alfo are wanting. GER. VOSSIUS, *de Scientiis Math.* cap. xvi. 7, when ftating the books of the *Mathematical Collections* adds, " fed duo primi videntur " deperiffe."

of the Parisian MSS. No. 2368, has this portion of the second book, beginning at the same words as the fragments in the Savilian MS.‡ From a MS. of PAPPUS, which belonged to JOSEPH SCALIGER, the portions of the preface to the viith book, describing the *Sectio Rationis* and *Sectio Spatii*, were published in Greek by SNELLIUS, in his restoration of these books, in 1607; and his edition of the Greek preface to the *Sectio Determinata*, 1608, was probably from the same MS.

The MS. of PAPPUS which was in the library of ISAAC VOSSIUS, and was carried from England to the University of Leyden, had also about one half of the second book*; and probably, the precise portion which is in the other MSS. now mentioned.

In the edition of the *Bibliotheca Græca*, published by HARLES, vol. ix. p. 170, is a catalogue of the MSS. of PAPPUS known to that writer, including those now mentioned and several others. I shall in this place therefore mention only two more, not taken notice of by him. In the year 1795, J. GUGL. CAMERER published an edition of the APOLLONIUS GALLUS by VIETA; and prefixes to it the preface and Lemmata belonging to the Tactions, in Greek. For this purpose he made use of three MSS. the two in the Parisian library No. 1440, and No. 2368; and another in the Strasburgh library, not mentioned in Fabricius; but from the very short references

‡ A MS. copy of this fragment of the second book, (with the third book also,) taken from this Parisian MS. is in the Savilian Library at Oxford; and from a note at the end of this copy, it appears that the Parisian MS. was written in 1562.

* See the account of ISAAC VOSSIUS in the *Biog. Brit.* This MS. is mentioned in the Oxford Catalogue of Manuscripts, No. 2126.

to that MS. nothing can be pronounced refpecting its hiftory and value.

There is alfo at Edinburgh an elegant manufcript of five books of PAPPUS, viz. the third, fourth, fifth, fixth, and eighth; but unfortunately the feventh, the moft valuable portion of the work, is wanting.

This MS. was purchafed at Paris, in 1748, by Dr. JAMES MOOR, then Greek profeffor at Glafgow, from Mr. DE MAIRAN, of the Academy of Sciences; and on the firft page of it is written *D'Ortous de Mairan*, probably by his own hand. DE MAIRAN had informed Dr. MOOR that it had belonged to BULLIALDUS; and on that account Dr. SIMSON, who had it for fome time in his poffeffion, and took many notes from it, calls it *Codex* BULLIALDI.† Like the other MSS. of PAPPUS it abounds with errors; but Doctor SIMSON obtained from it feveral improved readings and corrections, which he has remarked in his copy of COMMANDINE's tranflation. Some

† Dr. SIMSON, in his copy of COMMANDINE's PAPPUS, has the following notices of this MS. which, without doubt, he had from Dr. Moor; and which may therefore be confidered as authentic. In fol. 320. b. (PAPPI, edit. Pis. 1588.) "3tio Novris A. D. "1748. Predictam GREGORII (a Sanct. Vinc.) emendationem veram effe, oftendit "codex manufcriptus elegantiffimus, quem tribus abhinc diebus Parifiis huc attulit "D. JACOBUS MOOR, collega meus docftiffimus, &c."

Alfo in fol. 1. b. PAPPI, "Hifce litteris adfcriptis emendavimus tum fchema "COMMANDINI tum MSti GRÆCI quem Dom. *D'Ortous de Mairan* dicit fuiffe "BULLIALDI. R. S."

In a detached paper, referring to fol. 77. b. PAPPI, is a note in Dr. SIMSON's hand, "25 Martii 1750, inveni hæc ita effe in codice GRÆCO qui BULLIALDI fuerat, "quemque collega meus D. JACOBUS MOOR, a domino Mairan emptum Parifiis huc "(Glafguam) attulit."

of them are curious and important, and all merit the attention of any future editor of that work.

Some peculiarities in this MS. may be mentioned, for the ſatisfaction of thoſe who may be inclined to conſult it.

At the end of the third book are five pages of MS. not in COMMANDINE's edition, which contain ſome variations of the ſolution of the Deliacal problem, in addition to thoſe contained in the beginning of that book.§

At the end of the fourth book is a very imperfect ſketch of a Propoſition, not in COMMANDINE, which has already been mentioned in the preceding account of this fourth book.

In this MS. are many corrections on the margin, and ſome even in the text, by a later hand; and among them are a great number of the emendations propoſed by COMMANDINE of the very ſame errors, in the MS. uſed by him, from which a connection between theſe two MSS. may be inferred. Many of theſe emendations are ſervilely copied, retaining even the miſtakes into which COMMANDINE, in a few of them, had fallen.

This MS. though elegantly written, has been copied by a perſon totally ignorant of the ſubject; of which the number of groſs errors in its firſt ſtate is a ſufficient proof. It may be remarked, alſo, that at the beginning of a Propoſition, or of a paragraph, there is uſually a red letter, and a new line; but

§ To theſe additional pages the following title is prefixed, "ἄλλως.—το δέκατον "θεώρημα ἐν τῷ τρίτῳ τῆς τοῦ Πάππου συναγωγῆς καὶ την ἀπόδειξιν περιεχον, καὶ την ὀργάνικην "κατασκέυην τοῦ τε διπλασιασμοῦ τοῦ κύβου και τῶν δύο μεσῶν ἀνάλογον."—It may be obſerved that the SAVIL. MS. No. 3, has a ſimilar addition at the end of the 3d book.

frequently alſo this diſtinction is made in the middle of a ſentence, while the beginning of Propoſitions and ſubjects in other places is not diſtinguiſhed in that or in any other manner. The enumeration of the Propoſitions is often irregular, and many of the diagrams, though neatly drawn, are altogether erroneous. The MS. was ſold at Dr. MOOR's death, and was afterwards purchaſed for the library of the Faculty of Advocates at Edinburgh, in which the writer of this Memoir had every facility for conſulting that MS. and other books in that great collection. From the well-known liberality of the Curators of that eſtabliſhment, the aid of this MS. will, without doubt, be moſt readily given, when it can be uſeful for preparing an edition of that intereſting work in the original language.

This MS. not having the ſeventh book, Dr. MOOR procured a copy of it to be taken from the MS. No. 2368, in the Pariſian library. This alſo for ſome time was in Dr. SIMSON's poſſeſſion, and he adopts from it ſome corrections of COMMANDINE's tranſlation in his reſtoration of the *Loci plani*, and *Sectio determinata*; but, as was already mentioned, there is no reference to it in his expoſition of the Poriſms‡. I add with regret, that this copy, in the diſperſion of Dr. MOOR's library, ſeems to have been loſt; and theſe particulars are ſtated, as they may perhaps facilitate the recovery of it from ſome obſcure ſituation into which it may have accidentally fallen.

‡ In a note on COMMANDINE's PAPPUS, the Doctor gives the following account of this tranſcript, which he muſt have got from Dr. MOOR "Hunc autem librum 7mum PAPPI ex eo codice (ſcil. No. 2368, Reg. Bibl. Par.) deſcripſit Dom CAPERONIER, linguæ Græcæ in Academia Pariſienſi Profeſſor; ſumptibus D. JAC. MOOR, collegæ mei doctiſſimi. Schemata vero ejus depinxit D. JOA. BRISBANE, M.D."

The two Savilian MSS. No. 3, and No. 9, have been repeatedly referred to. There is alſo in that library a tranſcript from the Pariſian MS. No. 2368, of a portion of the ſecond book of PAPPUS, (the ſame as the fragment in MS. SAVIL. No. 9, publiſhed by Dr. WALLIS,) and the whole of the third book. This tranſcript is not mentioned by Dr. WALLIS; and in the following Appendix II. it is quoted as MS. SAVIL. B. From a note at the end, it appears that the Pariſian MS. No. 2368, was written in 1562, by the direction of P. RAMUS.

APPENDIX II.

TWO paſſages of PAPPUS, on the ancient diviſion of geometrical lines into claſſes, have, in the preceding Memoir, been repeatedly alluded to. For the ſatisfaction of thoſe who may wiſh to conſider them accurately, the Greek, from the SAVILIAN MSS. and the MS. BULL. is printed in this Appendix, along with COMMANDINE's tranſlation. Some of the various readings in theſe MSS. are remarked, but without taking notice of either the ſmaller differences, which do not affect the ſenſe, or of ſome groſs errors, manifeſtly ariſing from the careleſſneſs of the tranſcribers.

Some of COMMANDINE's notes are alſo mentioned; but as Dr. SIMSON has not left any obſervations on theſe paſſages, it is not my purpoſe to enter into any examination or explanation of them, except only by pointing out the references to ſome Propoſitions, not explicitly ſtated either in PAPPUS, or in the commentary of his tranſlator.

PAPPI, *MS.* SAVIL. *No.* 3. *Fol.* 10, *b*.

"Ἃ μὲν ἔδει με προειπεῖν ἐστι ταῦτα. Παρεὶς "δὲ κρίνειν σοί τε καὶ ἄλλοις τοῖς ἐν γεωμετρίᾳ "γεγυμνασμένοις τὰ ὑπ' ἐκείνου προσγραφέντα‖ "περὶ τῆς κατασκευῆς, καὶ τὰ ὑφ' ἡμῶν ἐπε-"νεχθέντα καλῶς ἔχειν ἡγοῦμαι, καὶ τὰ δόξαντα "τοῖς ἀρχαίοις περὶ τοῦ προειρημένου προβλή-"ματος ἐκθέσθαι. Καὶ πρῶτον εἰπεῖν ὀλίγα "περὶ τῶν ἐν γεωμετρίᾳ προβλημάτων ἀρχὴν "λαβὼν ἐντεῦθεν. Τῶν ἐν γεωμετρίᾳ προ-"βλημάτων οἱ παλαιοὶ τρία γένη φασὶν εἶναι. "Καὶ τὰ μὲν αὐτῶν ἐπίπεδα καλεῖσθαι, τὰ "δὲ στερεὰ, τὰ δὲ γραμμικά. Τὰ μὲν οὖν δι' "εὐθείας καὶ κύκλου περιφερείας δυνάμενα "λύεσθαι λέγοιτο ἂν εἰκότως ἐπίπεδα. Καὶ "γὰρ αἱ γραμμαὶ δι' ὧν λύεται τὰ τοιαῦτα "προβλήματα, τὴν γένεσιν ἔχουσιν ἐν ἐπιπέδῳ. "ὅσα δὲ προβλήματα λύεται παραλαμβανο-"μένης εἰς τὴν γένεσιν μιᾶς τῶν τοῦ κώνου "τομῶν, ἢ πλειόνων, ταῦτα στερεὰ κέκληται. "πρὸς γὰρ τὴν κατασκευὴν ἀναγκαῖόν ἐστι "χρήσασθαι στερεῶν σωμάτων† ἐπιφανείαις. "Λέγω δὲ ταῖς κωνικαῖς. Τρίτον δ' ἔτι κα-"ταλείπεται γένος, ὃ καλεῖται γραμμικόν.

"Quæ igitur me præmisisse oportebat, hæc sunt. Itaque omittens "explicare et tibi, et iis, qui in geo-"metria exercitati sunt, ea, quæ ille "scripsit de constructione, et quæ "nos objecimus; optimum fore ju-"dicavi, si exponerem quid antiqui "de dicto problemate senserint: et "primum nonnulla dicerem de pro-"blematibus, quæ in geometria con-"siderantur, inde sumpto initio.

"Problematum geometricorum an-"tiqui tria genera esse statuerunt, et "eorum alia quidem *plana* appellari, "alia *solida*, alia *linearia*. Quæ igitur "per rectas lineas, et circuli circumfe-"rentiam solvi possunt, merito *plana* "dicantur; etenim *lineæ*, per quas "ejusmodi problemata solvuntur, in "plano ortum habent. Problemata "vero quæcunque solvuntur, assumpta "in constructionem aliqua coni secti-"one, vel pluribus, *solida* appellantur; "namque ad constructionem necesse "est solidarum figurarum superfici-"ebus, nimirum conicis, uti. Restat "tertium genus, quod *lineare* appel-"latur. Lineæ enim aliæ præter jam

‖ προγραφέντα, MS. B. SAVIL.

† σχημάτων, idem.

" γραμμαὶ γὰρ ἕτεραι παρὰ τὰς εἰρημένας εἰς
" τὴν κατασκευὴν λαμβάνονται ποικιλωτέραν
" καὶ μεταπεπλασμένην§ ἔχουσαι τὴν γένεσιν.
" ὁποῖαι τυγχάν* (ουσι αἱ ἕλικες) καὶ τετρα-
" γωνίζουσαι καὶ κογχλοειδεῖς,‡ καὶ κισσοει-
" δεῖς, πολλὰ καὶ παράδοξα περὶ αὐτὰς ἔχουσαι
" συμπτώματα. Τοιαύτης δὴ τῆς διαφορᾶς
" τῶν προβλημάτων οὔσης, οἱ παλαιοὶ γεωμέτραι
" τὸ προειρημένον ἐπὶ τῶν δύο εὐθειῶν πρόβλημα
" τῇ φύσει στερεὸν ὑπάρχον, οὐχ οἷοίτε ἦσαν
" κατασκευάζειν τῷ γεωμετρικῷ λόγῳ κατα-
" κολουθοῦντες. ἐπεὶ μήδε τὰς τοῦ κώνου τομὰς
" ῥᾴδιο ἦν ἐν ἐπιπέδῳ γράφειν ἦν." [ὡς δεῖ δύο
" δοθεισῶν εὐθειῶν ἀνίσων, δύο μέσας ἀνάλογον
" λαβεῖν ἐν συνεχεῖ ἀναλογίᾳ] Τοῖς δὲ ὀργάνοις
" μεταλαβόντες αὐτὸ θαυμασίως εἰς χειρουργίαν
" καὶ κατασκευὴν ἐπιτήδειον ἤγαγον. ὥς ἐστιν
" ἰδεῖν ἀπὸ τῶν φερομένων αὐτοῖς συνταγμάτων.
" λέγω δ' ἐν τῷ ἐρατοσθένους μεσολάβῳ, καὶ
" τοῖς φίλωνος καὶ ἥρωνος μηχανικοῖς, ἢ κατα-
" παλτικοῖς. οὗτοι γάρ ὁμολογοῦντες στερεὸν
" εἶναι τὸ πρόβλημα, τὴν κατασκευὴν αὐτοῦ
" μόνον ὀργανικῶς πεποίηνται, συμφώνως ἀπολ-
" λωνίῳ τῷ περγαίῳ. ὃς καὶ τὴν ἀνάλυσιν
" αὐτοῦ πεποίηται διὰ τῶν τοῦ κώνου τομῶν.
" Καὶ ἄλλοι διὰ τῶν ἀρισταίου τόπων στερεῶν,

" dictas in constructionem assumun-
" tur, varium, et transmutabilem or-
" tum habentes, quales sunt helices,
" et [quas Græci τετραγωνίζουσαι appel-
" lant, nos] quadrantes [dicere possu-
" mus,] conchoides, et cissoides, qui-
" bus quidem multa, et admirabilia
" accidunt. Cum igitur tales sint pro-
" blematum differentiæ, antiqui geo-
" metræ problema ante dictum in
" duabus rectis lineis, quod natura
" solidum est, geometrica ratione
" innixi construere non potuerunt;
" quoniam neque coni sectiones facile
" est in plano designare: instrumentis
" autem ipsum in operationem manu-
" alem, et commodam, aptamque
" constructionem mirabiliter tradux-
" erunt, quod videre licet in eorum
" voluminibus, quæ circumferuntur,
" ut in ERATOSTHENIS mesolabo,
" in PHILONIS, et HERONIS me-
" chanicis et catapulticis. Hi enim
" asserentes problema solidum esse,
" ipsius constructionem instrumentis
" tantum perficerunt, congruentur
" APOLLONIO Pergæo, qui et resolu-
" tionem ejus fecit per coni sectiones:
" alii per locos solidos ARISTÆI:
" nullus autem per ea, quæ proprie

§ βεβιασμένην, idem.

* In MS. Savil. No. 3, quæ inter uncos, rubro liquore exaratæ a secunda manu.—In altero codice Savil. " αἱ ἕλικες" absunt. In MS. BULL. τυγχάνουσι αἱ . . . . καὶ.

‡ κογχοειδεῖς, MS. B. SAVIL.

‖ Quæ inter uncos, in COMMANDINI versione omittuntur; sed quamvis supervacanea, forsan non erronea.

" οὐδεὶς δὲ διὰ τῶν ἰδίως ἐπιπέδων καλουμένων.
" Νικομήδης δὲ καὶ λόγῳ διὰ κογχλοειδοῦς*
" γραμμικῆς, δι' ἧς καὶ τὴν γωνίαν ἐτριχοτό-
" μησεν. ἐκθησόμεθα οὖν τέσσαρας αὐτοῦ κατα-
" σκευὰς, μετά τινος ἐμῆς ἐπεξεργασίας. ὧν
" πρώτην μὲν ἐρατοσθένειον, δευτέραν δὲ τὴν
" τῶν περὶ νικομήδη, τρίτην δὲ τὴν τῶν περὶ
" ἥρωνα μάλιστα πρὸς τὰς χειρουργίας ἁρμό-
" ζουσαν, τοῖς ἀρχιτεκτονεῖν βουλομένοις. Καὶ
" τελευταίαν τὴν ὑφ' ἡμῶν ἀνευρημένην. στερεοῦ†
" γὰρ παντὸς, ἕτερον στερεον ὅμοιον τῷ δοθέντι
" κατασκευάζεται πρὸς τὸν δοθέντα λόγον.
" ἐὰν δύο τῶν δοθεισῶν εὐθειῶν δύο καὶ‡ μέσαι
" κατὰ τὸ συνεχὲς ἀνάλογον ληφθῶσιν, ὡς
" ἥρων ἐν μηχανικοῖς καὶ καταπαλτικοῖς.

" plana appellantur. At NICOMEDES, et ratione illud fecit per lineam conchoidem, per quam et angulum tripartito divisit. Exponemus igitur quatuor ejus constructiones una cum quadam nostra tractatione. Quarum prima quidem est ERATOSTHENIS, secunda NICOMEDIS, tertia HERONIS, maxime ad manuum operationem accommodata, iis qui architecti esse volunt. Ultima autem est a nobis inventa: solido enim quocunque dato, alterum solidum dato simile construitur ad datam proportionem, si duabus datis rectis lineis, duæ mediæ in continua analogia assumantur, ut inquit HERO in mechanicis et catapulticis.

* κογχοειδοῦς, MS. SAVIL. B.

† στερεοῦ δεδομένου. MS. BULL. et ut videtur MS. quo utebatur COMMANDINUS.

‡ Deest καὶ MS. B. SAVIL. et MS. BULL.

PAPPI, MS. Savil. No. 3. Fol. 86, b.

" Τὴν δοθεῖσαν γωνίαν εὐθύγραμμον εἰς τρία " ἴσα τεμεῖν οἱ παλαιοὶ γεωμέτραι θελήσαντες, " ἠπόρησαν, δι' αἰτίαν τοιαύτην. τρία γένη φαμὲν " εἶναι τῶν ἐν γεωμετρίᾳ προβλημάτων, καὶ τὰ " μὲν αὐτῶν ἐπίπεδα καλεῖσθαι, τὰ δὲ στερεὰ, " τὰ δὲ γραμμικά. τὰ μὲν οὖν δι' εὐθείας καὶ " κύκλου περιφερείας δυνάμενα λύεσθαι, λέγοιτ' " ἂν εἰκότως ἐπίπεδα. καὶ γὰρ αἱ γραμμαὶ " δὶ ὧν εὑρίσκεται τὰ τοιαῦτα προβλήματα " τὴν γένεσιν ἔχουσιν ἐν ἐπιπέδῳ ὅσα δὲ " λύεσθαι* προβλήματα, παραλαμβανομένης " εἰς τὴν γένεσιν μιᾶς τῶν τοῦ κώνου τομῶν, ἢ " καὶ πλειόνων, στερεὰ ταῦτα κέκληται. πρὸς " γὰρ τὴν κατασκευὴν χρήσασθαι στερεῶν σχη- " μάτων ἐπιφανείαις, λέγω δὲ ταῖς κωνικαῖς, " ἀναγκαῖον. τρίτον δέ τι προβλημάτων† ὑπο- " λείπεται γένος, τὸ καλούμενον γραμμικὸν, " γραμμαὶ γὰρ ἕτεραι παρὰ‡ τὰς εἰρημένας εἰς " τὴν κατασκευὴν λαμβάνονται, ποικιλωτέραν " ἔχουσαι τὴν γένεσιν, καὶ βεβιασμένην μᾶλλον, " ἐξ ἀτακτοτέρων ἐπιφανειῶν, καὶ κινήσεων ἐπι- " πλεγμένων γενόμεναι, τοιαῦται δέ εἰσιν αἱ τε " ἐν τοῖς πρὸς ἐπιφανείαις καλουμένοις τόποις " εὑρισκόμεναι γραμμαὶ, ἕτεραι τε τούτων

" Antiqui geometræ datum angulum " rectilineum tripartito secare volentes " ob hanc causam hæsitarunt. Pro- " blematum, quæ in geometria consi- " rantur, tria esse genera dicimus; et " eorum alia quidem *plana*, alia *solida*, " alia vero *lineuria* appellari. Quæ " igitur per rectas lineas, et circuli " circumferentiam solvi possunt, me- " rito dicuntur *plana*: *lineæ* enim per " quas talia problemata inveniuntur, " in plano ortum habent. Quæcun- " que vero solvuntur, assumpta in " constructionem aliqua coni sectione, " vel etiam pluribus, *solida* appellata " sunt, quoniam ad constructionem " solidarum figurarum superficiebus, " videlicet conicis, uti necessarium est. " Relinquitur tertium genus proble- " matum, quod *lineare* appellatur; " lineæ enim aliæ, præter jam dictas " in constructionem assumuntur, quæ " varium et difficilem ortum habent, " ex inordinatis superficiebus, et mo- " tibus implicatis factæ. Ejusmodi " vero sunt etiam lineæ, quæ in locis " ad superficiem dictis inveniuntur,

* λυθῆναι, MS. Savil. No. 9.   † των προβλημάτων, idem.

‡ περος, idem.

“ ποικιλώτεραι, καὶ πολλαὶ τὸ πλῆθος ὑπὸ
“ δημητρίου τοῦ ἀλεξανδρέως ἐν ταῖς γραμμικαῖς
“ ἐπιστάσεσι, καὶ φίλωνος τοῦ τυανέως, ἐξ ἐπιπ-
“ λοκῆς πλεκτοειδῶν§ τε καὶ ἑτέρων παντοίων
“ ἐπιφανειῶν εὑρισκόμεναι, πολλὰ καὶ θαυμαστὰ
“ συμπτώματα περὶ αὐτὰς ἔχουσαι. καὶ τινες
“ αὐτῶν ὑπὸ τῶν νεωτέρων ἠξιώθησαν λόγου
“ πλείονος. μία δέ τις ἐξ αὐτῶν ἐστιν, ἡ καὶ
“ παράδοξος ὑπὸ τοῦ μενελάου κληθεῖσα γραμμή.
“ τοῦ δὲ αὐτοῦ γένους ἕτεραι ἕλικές εἰσι τετρα-
“ γωνίζουσαί τε, καὶ κοχλοειδεῖς καὶ κισσοειδεῖς.
“ δοκεῖ δέ πως ἁμάρτημα τὸ τοιοῦτον οὐ μικρὸν
“ εἶναι τοῖς γεωμέτραις, ὅταν ἐπίπεδον προβλήμα
“ διὰ τῶν κωνικῶν ἢ τῶν γραμμικῶν ὑπό τινος
“ εὑρίσκεται. καὶ τὸ σύνολον ὅταν ἐξ ἀνοικείου

“ et aliæ quædam magis variæ, et
“ multæ a DEMETRIO ALEXAN-
“ DRINO [ἐν ταῖς γραμμικαῖς ἐπιστάσεσι,
“ hoc eſt] in linearibus aggreſſioni-
“ bus; et a PHILONE TYANEO ex
“ implicatione [πλεκτοειδῶν,*] et aliarum
“ varii generis ſuperficierum inventæ,
“ quæ multa et admirabilia ſympto-
“ mata continent: et nonnullæ ipſa-
“ rum a junioribus dignæ exiſtimatæ
“ ſunt, de quibus longus ſermo habe-
“ retur. Una autem aliqua ex ipſis
“ eſt, quæ et admirabilis a MENELAO
“ appellatur.

“ Ex hoc genere ſunt lineæ helices,
“ et quadrantes, et conchoides, et ciſ-
“ ſoides: videtur autem quodam-
“ modo peccatum non parvum eſſe
“ apud geometras, cum problema
“ planum per conica, vel linearia ab
“ aliquo invenitur, et ut ſummatim

§ πληκτοειδῶν. MS. BULL. et ut videtur MS. COMMAND.

* Πλεκτοειδων, or πληκτοειδων. This word, as appears from the application of it in Prop. 29. lib. iv. denotes a claſs of geometrical ſurfaces, produced by ſome preciſe rule, on which mathematical reaſoning reſpecting ſuch ſurfaces may be founded. By COMMANDINE, and alſo by VOSSIUS, they are ſuppoſed to have got this name from the complex motions by which they are produced. The *Plectoeides*, in Prop. 29, (ſee fig. of that Prop. in PAPPUS) is formed by the motion of the ſtraight line LKI paſſing through the ſtraight line BLN, to which it is always perpendicular, and alſo through the line produced by the common ſection of the conic ſuperficies, and of the ſuperficies called *Cylindroeides*, both mentioned in that Propoſition. See note G. p. 104 and 105.

Among the ancients, geometrical curves and figures generally got names from ſome reſemblance which they were ſuppoſed to have to common objects. Such are the *Conchoid* and *Ciſſoid*, and alſo the *Lunule*, or *Meniſcus*, the *Pelecoides*, the *Hippopeda*, and ſeveral others mentioned by PROCLUS. (See PROCL. on Def. 4. 8. 1 Elem.)——It is poſſible that πληκτοειδης may have had a ſimilar origin, not now to be traced; but the etymology implied in COMMANDINE's tranſlation of it may be right.

" γένους λύεται,|| διόν ἐστι τὸ ἐν τῷ ε̄ τῶν
" ἀπολλωνίου κωνικῶν, ἐπὶ τῆς παραβολῆς πρό-
" βλημα, καὶ ἐν τῷ περὶ τῆς ἕλικος ὑπὸ ἀρχι-
" μήδους λαμβανομένη στερ ὰ νεῦσις ἐπὶ κύκλου,
" μηδενὶ γαρ προσχρώμενον στερεῶ, δυνατὸν εὑρεῖν
" τὸ ὑπ' αὐτοῦ γραφόμενον ϑεώρημα. λέγω δὴ
" τὸ τὴν περιφέρειαν τοῦ ἐν τῇ πρώτῃ περιφορᾷ
" κύκλου, ἴσην ἀποδεῖξαι τῇ πρὸς ὀρθὰς ἀγομένῃ
" εὐθεία τῇ ἐκ τῆς γενέσεως τῆς ἐφαπτομένης
" τῆς ἕλικος. τοιαύτης δὴ τῆς διαφορᾶς τῶν
" προβλημάτων ὑπαρχούσης, οἱ πρότεροι γεω-

" dicam, cum ex improprio folvitur
" genere, quale eft in quinto libro
" conicorum APOLLONII‡ problema
" in parabola: et in libro de lineis
" fpiralibus ARCHIMEDIS: affumpta
" folida inclinatio in circulo: fieri enim
" poteft, ut nullo utentes folido, pro-
" blema ab ipfo defcriptum invenia-
" mus.† Dico autem circumferentiam
" circuli in prima circulatione defcrip-
" tam demonftrare æqualem rectæ
" lineæ, quæ a principio lineæ fpiralis
" ad rectos angulos ducitur ei quæ eft
" circulationis principium, et a recta
" linea fpiralem contingente termi-
" natur. Itaque cum hujufmodi fit
" problematum differentia, antiqui

|| λυῆ/ξαι γενεσις, MS. SAVIL. No. 9.

‡ In COMMANDINE's time, the firft four books only of APOLLONIUS's *Conics* had been difcovered; and he thence makes no obfervation on this criticifm of PAPPUS, refpecting a Propofition in the fifth book. ALEXANDER ANDERSON, however, in his *Exercitationum Mathematicarum Decas Prima*, (Paris 1619,) long alfo before the fifth, fixth, and feventh books of APOLLONIUS were recovered from the Arabic, inferred from inveftigations founded on the general defcription of that fifth book by PAPPUS, that the problem about the Parabola, here alluded to by him, was truly a folid problem; and in *Exerc.* V. (page 24,) he ftates it, and refolves it. In the fifth book of APOLLONIUS we now find the firft cafe of Prop. 58. is equivalent to the problem of ANDERSON.

† The Propofition of ARCHIMEDES mentioned in this place is the 18th in his treatife *De Helicibus*. In that Propofition the 7th is neceffary, and in that 7th Propofition (*De Helicibus*) the inclination alluded to is affumed.——COMMANDINE, in Note C. refpecting this inclination, refers to Prop. 128. b. i. of VITELLO's *Optics*. That Propofition is indeed a plane inclination, and is affumed in Prop. 5th of ARCHIMEDES's Treatife. But in Prop. 7. of the fame treatife, which (as has been obferved) is ufed in Prop. 18th, a folid inclination is affumed, the fame as Prop. 130. b. i. of VITELLO, in which an hyperbola is required. See alfo ALHAZEN, b. v. Propp. 32, 33. In Prop. 42 of b. iv. of PAPPUS, is a *Locus ad Hyperbolam*, which he ftates to have been affumed by ARCHIMEDES, in the folid inclination employed by him in his treatife *De Helicibus*.

" μετραι, τὸ προειρημένον ἐπὶ τῆς γωνίας πρό-
" βλημα, τῇ φύσει στερεὸν ὑπάρχον διὰ τῶν
" ἐπιπέδων ζητοῦντες, οὐχ οἷοι ἦσαν εὑρίσκειν,
" οὐδέ πω‖ γὰρ* τοῦ κώνου τομαὶ συνήθεις ἦσαν
" αὐτοῖς. καὶ διὰ τοῦτο ἠπόρησαν. ὕστερον
" μέντοι διὰ τῶν κωνικῶν ἐτριχοτόμησαν τὴν
" γωνίαν εἰς τὴν εὕρεσιν χρησάμενοι τῇ ὑπο-
" γεγραμμένῃ νεύσει."

" geometræ problema jam dictum in
" angulo, quod natura ſolidum eſt,
" per plana inquirentes invenire non
" potuerunt: nondum enim ipſis cog-
" nitæ erant coni ſectiones, et ob eam
" cauſam hæſitarunt. Poſtea vero
" angulum tripartito diviſerunt ex
" conicis, ad inventionem infra ſcripta
" inclinatione utentes.†

‖ ου δη πω, MS. Savil. No. 9.

* γαρ αἱ, MS. Bull. αι τε κωνε τομαι συνεχεις εισιν. MS. Savil. No. 9.

† Prop. 31, lib. iv. Pappi.

APPENDIX III.

AS the volume of Dr. SIMSON's posthumous works is not in general circulation, it may be agreeable to the Mathematical reader, to see the Doctor's very improved translation of the general description of EUCLID's Porisms, in the preface to the seventh book of PAPPUS, annexed to the account of his labours. The mutilated detail of the contents of the three books of Porisms, not being materially altered by Dr. SIMSON, except in a few Propositions restored by him, is not added.

"POST Tactiones in tribus libris habentur Porismata EUCLIDIS, collectio
"artificiosissima multarum rerum quæ spectant ad analysin difficiliorum et
"generalium problematum, quorum quidem ingentem copiam præbet natura.
"Nihil vero additum est iis quæ EUCLIDES primum scripserat, præterquam
"quod imperiti quidam qui nos præcesserunt secundas descriptiones paucis
"ipsorum [sc. Porismatum] apposuerunt. Cum vero unumquodque definitum
"habeat demonstrationum numerum,† ut ostendimus,‡ EUCLIDES unam
"eamque maxime evidentem in singulis posuit. Habent autem subtilem et
"naturalem contemplationem, necessariamque et maxime universalem, atque
"iis quæ singula perspicere et investigare valent admodum jucundam Specie

† "Forsan intelligit demonstrationes diversorum casuum ejusdem Porismatis; vel propo-
"sitionum quæ sunt ejus conversæ.

‡ "In quibusdam ex lemmatibus, ni fallor, ad Porismata.

" autem hæc omnia neque theoremata ſunt, neque problemata, ſed mediæ
" quodammodo inter hæc naturæ, ita ut eorum Propoſitiones poſſunt vel ut
" theoremata, vel ut problemata formari. Unde factum eſt, ut inter multos
" Geometras alii hæc genere theoremata exiſtiment, alii vero problemata,
" reſpicientes ad formam tantum propoſitionis. Differentias autem horum
" trium melius intellexiſſe veteres manifeſtum eſt ex definitionibus. Dixerunt
" enim theorema eſſe quo aliquid propoſitum eſt demonſtrandum; problema
" vero quo aliquid propoſitum eſt conſtruendum; Poriſma vero eſſe quo aliquid
" propoſitum eſt inveſtigandum. A Neotericis autem immutata eſt hæc
" Poriſmatis definitio, qui hæc omnia inveſtigare haud potuerunt, ſed
" Elementis hiſce adhibitis, oſtenderunt tantum quid ſit quod quæritur, non
" autem illud inveſtigaverunt. Et quamvis a definitione et ab ipſis rebus quæ
" traditæ ſunt redarguerentur, hoc tamen modo, ab accidente, definierunt.
" Poriſma eſt quod deficit hypotheſi a Theoremate Locali [hoc eſt, Poriſma eſt
" Theorema Locale deficiens ſive diminuta in hypotheſi ejus.] Hujus autem
" generis Poriſmatum Loca Geometrica ſunt ſpecies, quorum magna eſt copia
" in Libris de Analyſi; ac ſeorſim a Poriſmatibus collecta, ſub propriis titulis
" traduntur, eo quod magis diffuſa et copioſa ſit hæc præ cæteris ſpeciebus. E
" Locis enim quædam plana ſunt quædam ſolida, quædam linearia, et præter
" hæc ſunt Loca ad medietates [ſive a mediis proportionalibus orta.] Accidit
" hoc etiam Poriſmatibus, Propoſitiones habere conciſas propter difficultatem
" multarum rerum quæ ſubintelligi ſolent; unde evenit Geometras non paucos
" ex parte tantum rem perſpicere, dum ea quæ inter oſtenſa magis neceſſaria
" ſunt haud capiunt. Multa autem ex iis in una Propoſitione minime com-
" prehendi poſſunt, quia ipſe EUCLIDES non multa in unaquaque ſpecie
" poſuerit, ſed ut ſpecimen daret multæ copiæ, pauca ad principium primi
" libri poſuit ejuſdem omnino ſpeciei cum uberrima illa [quæ in primo libro
" habetur] Locorum ſpecie, ut decem ſint numero. Quare has [Propoſitiones
" ſcilicet hujus ſpeciei] una Propoſitione comprehendi poſſe animadvertentes,
" eam ita deſcribimus.

' Si quadrilateri cujus anguli oppoſiti vel ex adverſo, vel ad eaſdem partes
' ſunt poſiti,* [lateribus productis] data ſint in uno ipſorum tria puncta

* " Primum horum PAPPUS vocat ὕπτιον, alterum παρύπτιον, de quibus vide Not. C.
" pp. 85, 86, 87.

'[interfectionum scilicet;] vel si in quadrilatero cujus duo latera sunt inter se 'parallela [data sint duo puncta interfectionum in altera parallelarum;] cætera 'vero puncta præter unum tangant rectam positione datam; etiam hoc tanget 'rectam Positione datam.' "Hoc autem de quatuor tantum rectis dicitur "quarum non plures quam duæ per idem punctum transeunt. In quolibet "vero proposito rectarum numero ignoratur, quamvis vera sit hujusmodi "Propositio, viz.

'Si quotcunque rectæ occurrant inter se, nec plures quam duæ per idem 'punctum; data vero sint puncta omnia in earum una, unumquodque autem 'punctum in alia tangat rectam, positione|| datam.' Vel generalius sic. 'Si 'quotcunque rectæ occurrant inter se, neque sint plures quam duæ per idem 'punctum, omnia vero puncta [interfectionem scilicet] in earum unâ data sint; 'reliquorum numerus erit numerus triangularis, cujus latus exhibet numerum 'punctorum rectam positione datam tangentium; quarum interfectionum si 'nullæ tres existant ad angulos trianguli spatii [nullæ quatuor ad angulos 'quadrilateri, nullæ quinque ad angulos quinquelateri, &c. i. e. Universim, si 'nullæ harum interfectionum in orbem redeant] unaquæque interfectio reliqua 'tanget rectam positione datam.'

"EUCLIDEM autem hoc nescivisse haud verisimile est, sed principia sola "respexisse: nam per omnia Porismata non nisi prima principia, et semina "tantum multarum et magnarum rerum sparsisse videtur. Hæc autem juxta "hypothesium differentias minime distinguenda sunt; sed secundum diffe- "rentias accidentium et quæsitorum. Hypotheses quidem omnes inter se "differunt, cum specialissimæ sint: accidentium vero et quæsitorum unum- "quodque, cum sit unum idemque multis diversisque hypothesibus contingit.§

"Talia itaque inquirenda offeruntur in primi libri Propositionibus; (in "principio septimi habetur* diagramma huc spectans) 'Si a duobus punctis 'datis inflectantur duæ rectæ ad rectam positione datam, abscindat autem 'earum una à rectâ positione data segmentum dato in ea puncto adjacens, 'auferet etiam altera ab aliâ rectâ segmentum datum habens rationem.' i. e. "*Quod ad alterum segmentum habebit rationem eandem rationi quæ ex* "*hypothesi data est.*" Deinde in subsequentibus; &c."

|| "Unum tangat unam, aliud tangat aliam rectam positione datam, et sic deinceps."

§ "Ex gr. Multa sunt Porismata quæ diversas hypotheses habent, sed quæ omnia "concludunt punctum aliquod tangere rectam positione datam; vel rectam aliquam vergere "ad punctum datum, &c."

* "Non jam habetur."

ERRATA.

Page 18, line laſt of note, for 87 read 37
38, l. 6, note, for *rationum* read *rationem*
40, l. 5, note, for 280 read 270
89, l. 5, after *μέτα* place a period.
90, l. laſt of note, for γ read $\bar{\gamma}$ (3)
91, l. dele note, "*forſan ἡδομένοις*"
94, l. 1 note, for *indeterminatum* r. *indeterminatus*
114, l. 22, for *letter after*, read *letter, after*
121, l. 4, for *into ; it* read *into it ;*

Page 131, l. 3, for *treatiſes all of which* ; read *treatiſes, all of which*
132, l. 18, for *aſcertained* read *determined*
136, l. 5, note, for *Geometry* ; read *Aſtronomy*;
145, l. 1, note, for δ' ὅπῃ read δ', ὅπῃ
149, l. 12, for *plane*, read *plane*
170, l. laſt, for *ſame* read *fame*
175, l. laſt note, for *deperiſſe* read *deperiiſſe*
176, the firſt note, from page 180, unneceſſary.

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